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PRACTICAL MECHANICS

AND

ALLIED SUBJECTS

BY

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PREFACE

This book together with that on "Practical Applied Mathematics" by the same author embodies the results of five years' experience in the organization and development of a system of railroad shop apprentice schools, as well as extended investigations of the work of public, private, trade, vocational and corporation schools of manufacturing industries and railroads in this country and abroad.

In the presentation no formal distinction is made of the various branches of elementary mathematics and science involved. The subject matter is presented in the two volumes under the general titles of "Practical Applied Mathematics" and "Practical Mechanics and Allied Subjects," although a careful and logical arrangement and sequence have been followed in the presentation.

In organizing trade apprentice schools difficulty has been experienced in securing a text practically adapted to the needs of these schools. This difficulty has led the author to develop a series of instruction sheets which are believed to be thoroughly practical. This volume with that on "Practical Applied Mathematics" is essentially a presentation of problems arising in shop experience in the mechanical trades. It shows the use of mathematics as a tool.

The material included in this work has been made sufficiently broad to apply not only to railroad schools but it is believed to schools in the mechanical trades generally. It is also felt that this work may be used to

advantage in the public schools in technical branches, as well as in trade and vocational schools, either as a regular or as a supplementary text.

The material for each book is presented in twenty chapters, each dealing with a definite subject. This affords a flexibility highly desirable in assigning the work and enables it to be given entirely or in part according to specific needs.

In preparing this work the author acknowledges indebtedness to Pennsylvania Railroad Motive Power Officials at Altoona. The author is indebted to Dr. J. P. Jackson, State Commissioner of Labor and Industry of Pennsylvania, Professor J. A. Moyer, Head of the Department of Mechanical Engineering and Director of the Engineering Extension Division of the Pennsylvania State College and Mr. M. B. King, State Expert Assistant in Industrial Education of Pennsylvania, for reading the proofs and offering valuable criticisms and suggestions. He is especially indebted to Professor J. H. Yoder and Mr. E. W. Hughes of the School of Engineering of the Pennsylvania State College, detailed as Instructors in the Pennsylvania Railroad System of Apprentice Schools. They have assisted in the preparation of the manuscript, carefully read the proofs and offered many valuable suggestions.

JOSEPH W. L. HALE.

ALTOONA, PA.
June 10, 1915

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CHAPTER I

FORCES

1. Definitions.—In our every day work we deal with *forces* of one kind or another. A *pressure is a force.* The earth exerts a force of attraction for all bodies or objects on its surface. Steam exerts a force or pressure in a locomotive boiler and cylinder. In grinding tools a force of friction has to be overcome. The wind exerts a *force* or *pressure* against the surface which it strikes.

When forces act on objects they move the objects if the forces are great enough to overcome the resistance to motion, and even if the objects do not move the forces produce a *stress* and *pressure* within and on the objects.

Forces are measured in pounds and tons. When we consider forces acting on objects we must know how the forces are applied, the direction of the forces and their values in pounds and tons. In working out problems forces are often represented by straight lines. The length of the line represents to scale the value of the force as 100 lb., 20 tons, etc., and the direction of the line, the direction in which the force acts as vertical, horizontal, etc.

2. Forces Passing through a Point.—In Fig. 1 the line *Od* 1 in. in length represents a force of 15 lb. and the line *Ob* 2/3 of an in. in length represents a force of 10 lb. Also force *Ob* is horizontal and force *Od* acts at an angle of 60° with the horizontal.

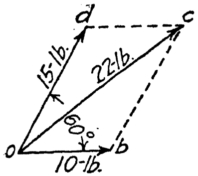


FIG. 1.

Two or more forces may be combined by the "parallelogram" method giving a single force having the same effect as the separate forces acting together, as for example in Fig. 1 forces of 10 lb. and 15 lb. acting on an object at 0 and 60° apart may be replaced by a single force *Oc* of 22 lb. which is the same in effect as the forces of 10 lb. and 15 lb. acting together. The ingle force *Oc* is called the "*resultant*" of the forces *Od* nd *Ob*. In this Fig. lines *Ob* and *dc* are parallel and equal, and the same is true of lines *Od* and *bc*.

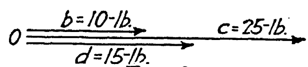


FIG. 2.

Rule.—The *resultant* of a number of forces is the direct sum of these forces *only* when they act in the *same straight line*, as in Fig. 2 and in *the same direction*.

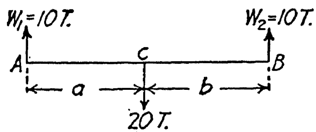


FIG. 3.

Any *force* has the greatest effect in the *direction* in which it acts, and *no effect* in a *direction at right angles* to that in which it acts.

3. Parallel Forces.—Parallel forces are those which act in directions parallel to one another and which do not meet. In Fig. 3 is shown three forces, one of 20 tons and two of 10 tons each, balancing the *load* of 20 tons. For *parallel forces* as in this Fig. $W_1 + W_2 = 20$, and

$$\frac{W_1}{20} = \frac{b}{a + b}$$

and

$$\frac{W_2}{20} = \frac{a}{a + b}$$

PROBLEMS

1. What is the total force or pressure acting on a 14-in. piston in a steam cylinder with a steam pressure of 180 lb. per sq. in.?

2. What must be the diameter of a piston in an air cylinder for a total pressure of 3000 lb. with a pressure per sq. in. of 60 lb.?

3. If the total pressure produced on a piston by steam in an 18-in. cylinder is 45,600 lb., what is the steam pressure in lb. per sq. in.?

4. What force of attraction does the earth produce on a steel rod 4 in. in diam. and 3 ft. 6 in. in length?

5. Two forces of 20 lb. and 30 lb. act on an object in opposite directions. What is the "resultant" force? If these forces acted at right angles to one another what would be the "resultant" force?

6. What is the total pressure produced on the supporting structure for a water tank if the tank is 12 ft. high and 8 ft. inside diam. and full of water? The material of the tank weighs 1280 lb.

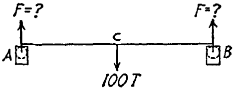


FIG. 4.

7. A traveling crane (Fig. 4) lifts a 100-ton locomotive. With the load half way between the crane supports A and B, what pressure is brought on each of these supports? How is this pressure changed when the load c moves nearer end A?

8. Using Fig. 4, find the pressures produced on supports at both A and B with the following loads and distances of the loads from A and B.

	Load at C	Distance AC	Distance BC	Pressure at A	Pressure at B
1	20 tons	20 ft.	40 ft.		
2	15 tons	32 ft.	28 ft.		
3	100 tons	10 ft.	50 ft.		
4	2500 lb.	50 ft.	10 ft.		
5	30 tons	42 ft.	18 ft.		
6	390 lb.	25 ft.	35 ft.		

9. What steam pressure is used on locomotives? Is the same pressure used for heating cars and the shops?

Give complete answer with reasons.

10. If the steam pressure is 30 lb. per sq. in., what is the total force tending to blow the end out of a steam pipe 4 in. inside diam.?

CHAPTER II

GRAVITATION. CENTER OF GRAVITY

4. Definition of Gravity.—The attractive force which every object in the universe has on every other object is called "gravitation" and this depends on the quantity of material composing the objects.

The force of attraction of the *earth* for objects is called the *force of gravity* and is what gives objects their "*weight*." A spring balance in giving the "weight" of any object measures merely *the force of attraction of the earth for that object.*

On account of "the force of gravity" which tends to pull objects toward the center of the earth work has to be done in lifting them and for the same reason elevated objects have power to do work on account of their elevated position, as in the case of water above a fall, elevated weights, train on a down grade, lifted hammer, etc. A locomotive hauling a train up a grade consumes more coal than when the train is on level track since work has to be done in lifting the weight of the train against the force of gravity as well as overcoming frictional resistances. A train descending a grade has work done *on it* by the force of gravity. The train will *coast* and often attain a high speed without the use of any steam.

5. Gravity Action On Freely Falling Objects. The "force of gravity" acting on objects free to fall increases their speed about 32.2 ft. per sec. each second of the

fall. For example, a weight starting from rest and falling freely will have a speed of 96.6 ft. per sec. at the end of the third second of its fall and 161.0 ft. per sec. at the end of the fifth second of its fall.

6. Work Done Against Gravity.—The work done in lifting objects against "the force of gravity" is measured in *foot-pounds* and is equal to the weight of the object in pounds multiplied by the vertical height in feet through which the object is lifted.

PROBLEMS

1. A train is made up of a locomotive weighing 272,000 lb., a 158,000-lb. tender, a baggage and mail car weighing 86,000 lb. and 5 Pullman sleepers weighing together 625,000 lb. How much work is required to lift the train up a mile of grade rising 1 ft. in 250 ft.?

2. How much work in foot-pounds can a 2-ton hammer do in each fall if it is raised 16 ft.?

3. How much work in foot-pounds is required to load the following lot of tires on a flat car, if the average distance through which they are raised is 6 ft.?

50 tires each weighing 452 lb.
37 tires each weighing 643 lb.
24 tires each weighing 598 lb.
16 tires each weighing 544 lb.

4. If a 10-ton casting is raised 23 ft. by a traveling crane, how much work is done? If one horsepower is the performance of 33,000 ft.-lb. of work per min. and the above casting is raised in 1 min., what horsepower does the motor on the crane exert?

5. Using the definition for "horsepower" given in Prob. 4, what work in foot-pounds could a 10-h.p. engine do in 5 min. if it ran at full capacity for this time?

6. A single acting pump raises 1600 gal. of water every minute through a distance of 5 ft. How much work against gravity does the pump do per hr.?

7. A train weighing 1000 tons descends 2 miles of grade dropping

1 ft. in 200 ft. What work in foot-pounds does "gravity" do on the train in the descent?

8. How much work is done per hr. in raising a 1-ton hammer through a vertical height of 4 ft. an average of fifteen times per min.?

9. If an object falls freely for 7 sec., what is its speed at the end of this time? Would a bullet and a feather fall side by side in the air? Has air friction any effect on the speed of a falling object?

7. **Definition of Center of Gravity.**—In the fourth paragraph we learned that the earth attracts toward its center every particle of every object on its surface, and this force of attraction called the *force of gravity* gives objects their weight. When solving problems in mechan-

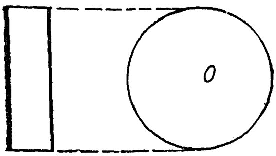


FIG. 5.

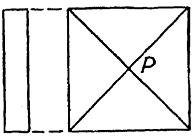


FIG. 6.

es it is often necessary to find a point in an object at which its entire weight may be considered as concentrated. This point about which an object would balance is called its *center of gravity*. For example, a circular steel plate as in Fig. 5, has its "center of gravity" at the center 0, half way between the faces of the plate. If the metal in this plate weighed .30 lb. per cu. in. and the plate contained 100 cu. in. of metal we could consider its entire weight of .30 × 100 or 30 lb. as acting at 0, instead of taking the weight of each cu. in. separately. This gives only one value, 30 to work with instead of 100 values, each of .30.

The entire weight of a square uniform metal plate as in Fig. 6 may be taken at a point P at the intersection of the diagonals of the square and half way between the faces of the plate. The center of gravity of a cylinder is at a point on the axis of the cylinder and half way between the upper and lower bases.

The position of the center of gravity of an object such as a metal sheet, as shown in Fig. 7, can be found experimentally by using a plumb line and a support on which the object can move freely. When an object is suspended from a support on which it can move freely the "center of weight" or "center of gravity" is brought as low as possible, and therefore lies in a vertical line drawn through the point of support. Using this principle an object as a steel plate shown in the figure is suspended freely from the support "s" and a plumb line dropped from the point of support as shown. The direction of the line is then marked on the plate giving line "sa."

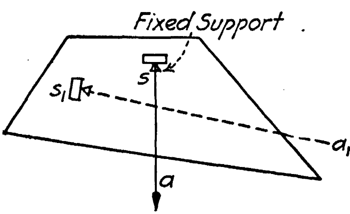


FIG. 7.

The plate is next supported from a second point and with the plumb line we get line s_1a_1. The point where lines sa and s_1a_1 cross marks the position of the "center of gravity" of the plate, half way between the front and back faces, if the plate is of uniform thickness throughout.

The center of gravity of sections made up of a number of simple sections as shown in Fig. 8, composed in this case of three rectangles, is found as follows: First we choose two "reference axes" as lines ab and cd crossing at 0. Next we find the *product* of the area of each simple

figure and the distance of *its* center of gravity from the chosen axis, then we add these products together and divide their sum by the sum of the areas of the simple figures.

For example, in the figure shown, the area of rectangle number 1 is $2 \times 1\frac{1}{2}$ or 3 sq. in. The area of No. 2 is 6×3 or 18 sq. in. and of No. 3, 2×2 or 4 sq. in. The

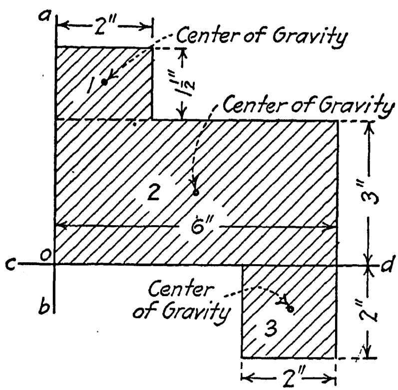


FIG. 8.

sum of the products of these areas by the distances of their respective centers of gravity from axis *ab* gives $3 \times 1 + 18 \times 3 + 4 \times 5$ or 77. Dividing this by the sum of the areas of the simple figures we have

$$\frac{77}{3 + 18 + 4} \text{ or } \frac{77}{25} \text{ or } 3.08 \text{ in.}$$

which is the distance of the center of gravity of the whole figure from line or axis *ab*.

Working in the same way from axis *cd* we have the equation

Distance of *c.g.* from line

$$cd = \frac{3 \times 3.75 + 18 \times 1.5 - 4 \times 1}{3 + 18 + 4}$$

or $\frac{34.25}{25}$ which equals 1.37 in.

The point where the lines cross as indicated in Fig. 9 is the position of the center of gravity of the figure or the point on which the whole plate would balance. In the equation above the last term in the numerator that

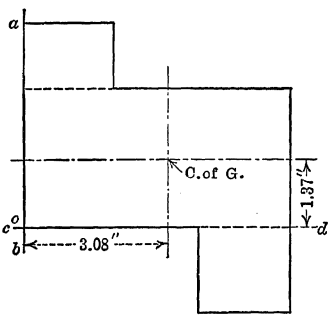


FIG. 9.

is 4×1 has to be subtracted as shown rather than added since the area 4 sq. in. is on the *other side* of the reference axis *cd*, and if this was, for example, a metal plate we were calculating, the weight represented by the lower rectangle would help counterbalance the weight of the parts on the other side of the axis *cd*.

Example.—Find the center of gravity of the metal plate shown in Fig. 10 in which are cut two holes 3 in and 6 in. in diam.

Solution.—Distance of center of gravity from axis *ab* equals

$$\frac{18 \times 15 \times 9 + 4 \times 3 \times 20 - 6^2 \times .785 \times 7 - 3^2 \times .785 \times 14}{18 \times 15 + 4 \times 3 - 6^2 \times .785 - 3^2 \times .785}$$

or $\frac{2373.3}{246.68}$ which equals 9.62 in.

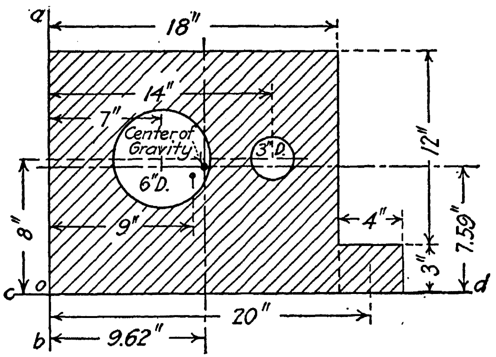


FIG. 10.

Distance of center of gravity from axis *cd* equals

$$\frac{18 \times 15 \times 7.5 + 4 \times 3 \times 1.5 - 6^2 \times .785 \times 8 - 3^2 \times .785 \times 8}{18 \times 15 + 4 \times 3 - 6^2 \times .785 - 3^2 \times .785}$$

or $\frac{1873.4}{246.68}$ which equals 7.59 in.

PROBLEMS

10. Find the center of gravity of the angle iron section shown in Fig. 11.

11. Find the center of gravity of the "Z" beam section shown in Fig. 12. If this beam is 10 ft. long, find its weight at .28 lb. per cu. in. for steel.

12. Find the center of gravity of the I-beam section shown in

Fig. 13, and its weight if 18 ft. long and steel weighs .28 lb. per cu. in.

8. Stability of Objects.—Whether an object will stand up or fall over, that is, its "stability," depends upon

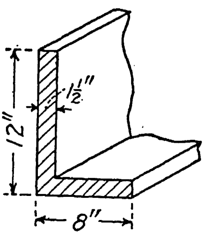


FIG. 11.

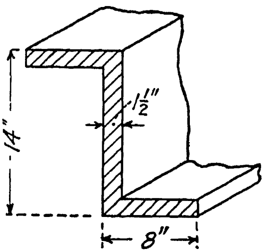


FIG. 12.

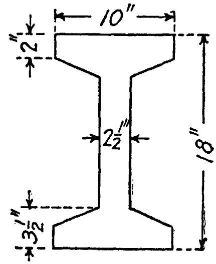


FIG. 13.

the position of its "center of gravity" with respect to its base. The rule is as follows: If a *vertical* line through the "center of gravity" passes inside the base of an object as in Fig. 14, the object will not fall, but if the

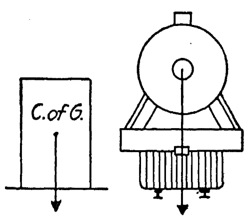
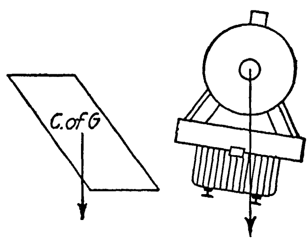


FIG. 14.



FIGS. 15 (*a*) and (*b*).

vertical line comes outside the base as in Fig. 15*a*, the object will fall. From this it follows that objects which are heavy and have a large base are less likely to fall, than those which are lighter and have a smaller base,

and also the *lower* the "center of gravity" in an object the less likely it is to fall. The center of gravity of an "area" or "surface" of any kind is understood to mean that point about which a very thin sheet representing the surface would balance. Fig. 15*b* representing the end of a locomotive shows the change in position of the "center of gravity" with respect to the base when the locomotive is on a depressed rail on a curve.

PROBLEMS

13. A steel plate 2 ft. $\times$ 4 ft. is $\frac{3}{8}$ in. thick. What is its weight and at what point in the plate can this entire weight be considered as concentrated? What is this point called?

14. Where is the "center of gravity" of a sphere located?

15. At what point in a steel bar 4 in. in diam. and 6 ft. long can the entire weight be considered as acting? What is this weight?

16. Where is the center of gravity of a cube located? Find the weight and center of gravity of a cube of steel 2 in. on a side.

17. Why is it desirable to have the center of gravity of a pulley in line with the center line of the shaft on which the pulley works?

18. How could you find the center of gravity of any triangularly shaped piece of steel plate?

19. Is it necessary to use a counterbalance on a fly-wheel, and if so explain why?

CHAPTER III

DENSITY AND SPECIFIC GRAVITY

9. Definitions and Problems.—The *density* of an object means its mass or quantity of matter per unit of volume. Suppose a cubic foot of metal weighs 450 lb., we may then say that the density of the metal is 450 lb. per cu. ft. Since all objects are not of the same weight it is found convenient to have a ready means for calculating their relative weights. In order to do this, tables are prepared which give us the weights of unit volumes of materials compared with the weight of the same volume of some material taken as a standard.

Specific Gravity is the ratio of the density of two objects. For example: If a cubic foot of metal weighs 490 lb. and it is compared with the weight of a cubic foot of water which is about 62.5 lb., the ratio is $\frac{490}{62.5}$ or 7.83 which is the specific gravity of the metal referred to water.

If a block of granite containing 1 cu. ft. volume weighs 162 lb., its weight compared to a cubic foot of water is $\frac{162}{62.5}$ or 2.59 which is its specific gravity referred to water as a standard. In the same way we can compare the weights of other materials with water and prepare a table which we can use to advantage in calculations involving the weights of the materials most used.

In any standard handbook may be found such tables

already prepared for a large number of substances. In making out these tables the weights of the bodies if they are solid or liquid are compared to the weight of the same volume of water as a standard, the weight of the water being taken as that at its temperature of greatest weight, namely, 4° Cent. If we are finding the relative weight of a gas we use *hydrogen gas* as a standard.

The *specific gravity* of a substance is therefore the ratio of the weight of a given volume of the substance to the weight of a similar volume of a substance taken as a standard. The specific gravity of a substance is therefore merely a *number* or a *ratio* and has no name attached to it.

If W is the weight of a substance, S the weight of the same volume of a standard substance and g the specific gravity of the substance we have the rule $g = \frac{W}{S}$. From this rule $W = g \times S$, that is, the weight of the substance is equal to its specific gravity times the weight of the same volume of the standard substance. Also we can write the equation in the form $S = \frac{W}{g}$.

It is known that an object entirely immersed in a liquid loses in weight an amount equal to the weight of the volume of the liquid displaced by the object immersed. This principle enables us to determine easily the specific gravity of some substances by completely immersing them in liquids in which they are not soluble.

Example.—As shown in Fig. 16, a small cast-iron cylinder weighs in air 1.12 lb. When completely immersed in water as shown its weight is only .965 lb. The loss in weight of the object due to submerging it in

water is therefore .155 lb. and its specific gravity equals

$$\frac{\text{weight in air}}{\text{loss of weight when immersed in water}} = \frac{1.12\text{ lb.}}{.155\text{ lb.}} \text{ or } 7.22.$$

Evidently we could not use this method with water as a liquid if we wished to find the specific gravity of sugar, since sugar is soluble in water. In the case of substances which are dissolved by water we may use other liquids as the standard by which the substance is not dissolved and compare the weight of this new liquid with the weight of water in determining the specific gravity.

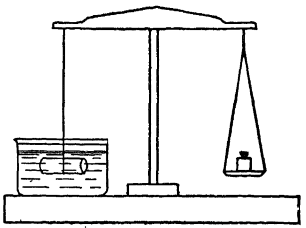


FIG. 16.

Below is given a table of the approximate specific gravities of some of the most common materials.

APPROXIMATE SPECIFIC GRAVITIES

Iron, cast....	7.21	Copper......	8.79	Granite......	2.6
Iron, wrought	7.78	Nickel.......	8.8	Glass........	2.9
Steel........	7.92	Mercury, at 60°	13.6	Pine, white..	.55
Lead........	11.3	Gold........	19.26	Pine, yellow .	.66
Tin.........	7.29	Silver.......	10.47	Oak, white...	.77
Zinc........	7.19	Aluminum...	2.67	Marble......	2.7
Brass.......	7.82	Cork........	.24	Ice..........	.917

Example.—Using the above table suppose we wish to find the weight of an iron casting containing 2½ cu. ft.

Solution.—One cubic foot of water weighs 62.5 lb. The specific gravity of cast iron is 7.21, that is, the cast iron is 7.21 times heavier than water, therefore the weight of the casting equals

$$62.5 \times 2.5 \times 7.21 \text{ or } 1126.56\text{ lb.}$$

Example.—What is the weight of a white-oak beam 4 in. × 8 in. and 18 ft. long?

Solution.—We first find the volume of the beam in cubic feet which is equal to $\frac{4}{12} \times \frac{8}{12} \times 18$ or 4.0 cu. ft.

Since from the table we see that the specific gravity of white oak is .77 that is, since it is .77 times as heavy as water the weight of the beam equals

$$\text{cubic feet} \times 62.5 \times .77$$

that is $4.0 \times 62.5 \times .77$ or 192.5 lb.

Example.—A timber is thrown into still water and it is found that it floats one-third out of water, what is its specific gravity?

Solution.—Since the timber displaces a volume of water equal to $\frac{2}{3}$ of its own volume it is held up or buoyed up by a force equal to the weight of a volume of water $\frac{2}{3}$ that of the volume of the timber. The weight of the timber is therefore only $\frac{2}{3}$ of the weight of the same volume of water, that is, its specific gravity is .67.

Example.—What is the volume of 100 lb. of copper?

Solution.—One cubic inch of water weighs $\frac{62.5}{1728}$ or .0361 lb. Since the specific gravity of copper is 8.79, 1 cu. in. of copper weighs $8.79 \times .0361$ or .317 lb. There are therefore as many cubic inches of copper in 100 lb. as .317 lb. the weight of 1 cu. in. of copper is contained into the total weight of copper or $\frac{100}{.317} = 315.4$ cu. in.

PROBLEMS

1. Using the table of specific gravities, find the weight of 8 cu. ft. of wrought iron.

2. Find the weight of a white pine timber 4 in. × 6 in. and 14 ft. long.

3. How many pounds heavier is a cubic foot of lead than a cubic foot of wrought iron?

4. Find the number of cubic feet in a marble block weighing 2½ tons.

5. A cylindrical casting 2½ in. in diam. and 8 in. long weighs 11.0 lb. What is its specific gravity?

6. A tank car has a capacity of 50,000 lb. If it is to be filled with petroleum having a specific gravity of .832, how many cubic feet and how many gallons can the tank hold?

7. The pressure of the atmosphere per square inch is about 14.7 lb. How high a column of mercury 1 sq. in. in cross section is required to produce this pressure?

8. A steel casting displaces 4.84 gal. of water. What is its weight?

9. Two hundred cubic inches of brass are made up of 150 parts of copper and 50 parts of zinc by volume. Find its specific gravity.

Solution:

The weight of 1 cu. in. of water = .0361 lb.

The weight of 1 cu. in. of copper = $8.79 \times .0361$ or .317 lb. Therefore 150 cu. in. weigh 47.6 lb. The weight of 1 cu. in. of zinc = $7.19 \times .0361$ or .260 lb. Therefore 50 cu. in. weigh 13.0 lb.

If S is the specific gravity of the brass, 200 cu. in. weigh

$$200 \times .0361 \times S \text{ lb.}$$

Since the weight of the brass equals the sum of the weights of its parts, we have, $200 \times .0361 \times S = 47.6 + 13$

therefore

$$S = \frac{47.6 + 13}{200 \times .0361} \text{ or } 8.39 \text{ the specific gravity.}$$

10. Forty pounds of copper and 27 lb. of zinc are alloyed to make brass. What is its specific gravity?

11. A piece of metal weighs 26.8 lb. in air and 23.1 lb. when entirely immersed in water. What is its specific gravity?

12. Find the weight of oil in a cylindrical tank 8 ft. in diam. and 16 ft. long if the specific gravity of the oil is .98.

13. A casting weighs in air 22.8 lb. When entirely immersed in oil whose specific gravity is .94 the casting weighs 19.6 lb. Find its specific gravity and its volume in cubic inches.

14. An alloy is made up of 30 cu. in. of tin and 48 cu. in. of copper. Find its specific gravity.

CHAPTER IV

SCREW THREADS

10. Thread Sections.—The shape and size of a thread are determined by the shape and size of its cross section taken in a plane passing through the axis of the screw. The dimensions of any particular kind of thread are fixed by stating the number of threads or sections in an inch, measured parallel to the axis of the screw, as, for example, 8 threads per inch, or simply 8 threads.

Fig. 17 shows the sections of four different screw threads. In the machine shop on regular work the U. S.

FIG. 17.

Std. Thread is used. In some practice bolts and studs ¾ in. in diam. and above screwed into boilers have twelve sharp V threads per in., irrespective of the diameter of the bolt or stud.

11. Pitch and Lead.—Before looking into the method of constructing and calculating threads we should understand first the meaning of the "pitch" and "lead" of a screw. The "pitch" of a screw is the distance from the center of one thread to the center of the next. The "lead" of a screw is the distance the screw advances in one complete turn.

12. Single, Double, and Triple Threads.—A single-threaded screw, as shown in Fig. 18, is one which advances in one complete turn a distance (measured in the direction of the axis of the screw) equal to that from a point on one thread to a corresponding point on the next thread, or a distance equal to the "pitch."

A double-threaded screw, as shown in Fig. 19, ad-

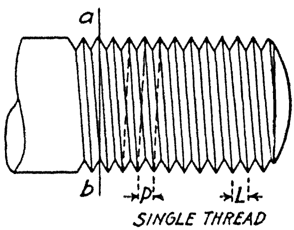


FIG. 18.

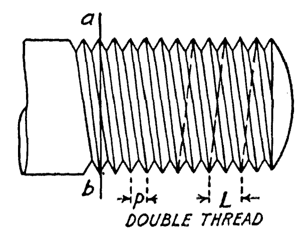


FIG. 19.

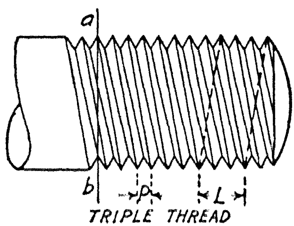


FIG. 20.

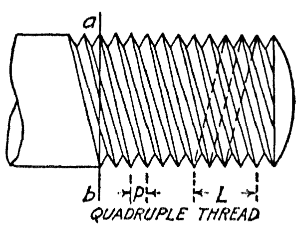


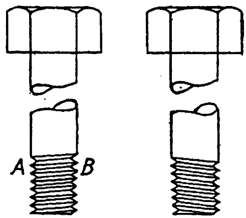
FIG. 21.

vances in one complete turn a distance equal to twice the "pitch." For a double thread the "lead" is therefore twice the "pitch." In a triple thread, as shown in Fig. 20, the advance for one complete turn equals three times the "pitch," or the "lead" equals three times the "pitch." Fig. 21 shows a quadruple thread.

13. Right- and Left-handed Threads. A right-handed thread is one in which the thread rises from left to right,

as from A to B in Fig. 22. Fig. 23 shows a left-handed thread.

Suppose now, for example, we were making a sketch of a threaded bolt, we should show the following. The total length of bolt including the head, the length of body including the threaded portion, length of threaded portion, diameter of bolt, the thread section, number of threads per in., whether thread is right- or left-handed and whether single, double or triple. If there are no standard tables available for finding the dimensions of the bolt head, the thickness and short diameter of the head should be shown.



FIGS. 22 and 23.

14. Construction of a Single, Right-handed V Thread, 8 Threads to an Inch.—Lay off on line AB, Fig. 24 (an element of the portion to be threaded), ⅛-in. distances as 1–2, 2–3, etc., each equal to the pitch of the screw. With the 60° triangle draw the sections 1a2, 2b3, etc. A line through point a parallel to line AB contains the points b, c, d, etc., at the bottoms of the threads. A perpendicular from point a to line CD gives point 5 on the *outside* of the thread. Similarly perpendiculars dropped from points b, c, d, etc., locate points on the outside of the thread on line CD. With the 60° triangle the sections 5e6, etc., are completed and the visible edges of the thread are drawn by connecting points b and e , 2 and 5, etc., as

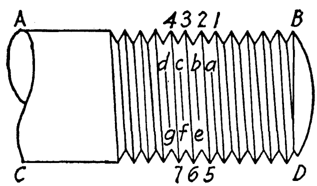


FIG. 24.

shown. The edges of the thread form turns called "helices," a single turn being a "helix." These edges are, however, simply represented by straight lines which are used in most cases as in Fig. 24.

15. Construction of a Double, Left-handed V Thread, 4 Threads to an Inch.—In Fig. 25 portion *AB* is to be threaded. The "pitch" is here ¼ in., and the "lead" ½ in. Since the thread is *double*, the outside points of the thread on opposite sides of the screw fall opposite each other. The construction is otherwise the same as for the single V thread. In one complete turn the screw advances a distance equal to twice the "pitch," or a distance of ½ in. equal to the "lead" of the screw. The distance *BE* corresponds to the pitch. The screw is left-handed, since the thread ascends from right to left looking toward the head of the bolt.

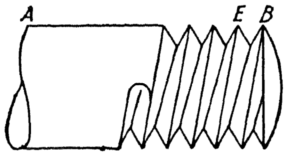


FIG. 25.

16. Construction of a United States Standard Thread.—The U. S. Std. for screws and threads is that adopted by the U. S. Government and by the R. R. Master Mechanics and Master Car Builders Associations, also by leading tool builders and manufacturers. This thread has the same angle as the V thread or 60° thread. The dimensions of a thread for a pitch equal to 1 in. are shown in Fig. 26. The triangle *ABC* is equilateral. If each of its sides equals 1 in. the distance *CD* equals the square root of $[(1)^2 - (\frac{1}{2})^2]$ or the square root of (1 − 0.25) or 0.866 in. This is therefore the depth of a V thread of pitch equal to one. In this U. S. Std. thread the depth *EF* = 0.65 in., that is, for a pitch of 1 in. The distance across the flats at top and bottom is 0.125

in., as shown in the figure, that is, 1/8 of the pitch. To find the depth of a U. S. Std. thread of another pitch, divide these figures by the number of threads to the inch.

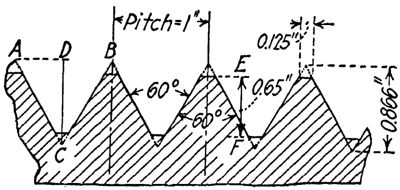


Fig. 26.

PROBLEMS

1. What is the lead of a single-threaded screw having 12 threads per in.? What is the pitch of this screw?

2. A double thread measures .125 in. between the points, what is its pitch and lead?

3. How many turns must a single thread of pitch .166 in. make to advance 1¼ in.?

4. A double-threaded screw advances 1½ in. in three complete turns. What is its pitch?

5. Find the depth of thread, and width of flats for a U. S. Std. Thread, with the following number of threads per in. 8, 12, 13, 18.

6. The number of threads per inch (U. S. Std.) corresponding to the diameter of the bolt are as follows:—Diam. of bolt ½ in.; threads per in., 13; diam. 7/8 in.; threads, 9; 1¼ in. diam. threads, 7. For each of these cases calculate the diameter at root of thread and the area of bolt at root of thread, also the width of flat.

7. In a machine, three bolts each 1 in. in diam. with 8 U. S. Std. threads per in., are replaced by a single bolt. What must be the size of this bolt at root of thread to have the same area of metal at root of thread as the three 1-in. bolts?

8. What is the "pitch" and the "lead" of a triple thread which advances 2 in. in two complete turns?

9. Make a sketch of a double, right-hand V thread with a pitch of 1/8 in.

10. Find the size of a tap drill in nearest 64ths in. for a "V" thread 8 per in. on a bolt 1 in. in diam.

11. Make a sketch of a single right-hand U. S. Std. thread with 8 threads per in. What is the pitch and the lead in this case?

12. How does the number of threads per in. on a bolt and hence the depth of the thread affect the strength of the bolt? Would it be practicable to have only 4 threads per in. on a 1-in. bolt? Why with some companies are 12 sharp V threads per in. used in boiler work on any size bolt or stud above $\frac{3}{4}$ in. in diam.?

13. What is the advantage of double and triple threads over a single thread? How does the number of threads per in. affect the "speed" of a bolt or screw? Which form of thread is used most in the machine-shop work, the V or U. S. Std.?

Referring to Fig. 27, showing the section of a V thread, the distance AB, or the diameter at the root of the thread equals the diameter of the bolt minus twice the distance CD. The length CD is .86 of the distance EF, and the distance EF depends on the number of threads per in. of the screw. In an 8-thread screw $EF = \frac{1}{8}$ in. or .125 in., for a 10-thread screw $EF = \frac{1}{10}$ in. or .10 in., for a T-thread screw $EF = \frac{1}{T}$.

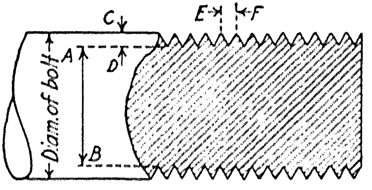


FIG. 27.

From the above we have the rule for a V thread.

$$R = D - \frac{1.73}{T}$$

where R = Diam. at root of thread.
D = Diam. of bolt (outside of threads).
T = No. of teeth or threads per in.

If in the case of bolts or studs $\frac{3}{4}$ in. in diam. and above, screwed into boilers, 12 sharp V threads per in. are used,

of the diameter of the bolt or stud, the pre- becomes in this case:

$$R = D - \frac{1.73}{12} \text{ or } R = D - 0.144 \text{ in.}$$
$$\text{or } R = D - {}^{9}\!/_{64} \text{ in.}$$

e of the U. S. Std. thread as in Fig. 28 the t the root of the thread equals the diameter t minus twice the distance AB. The distance × the pitch. Therefore the diameter at the e thread

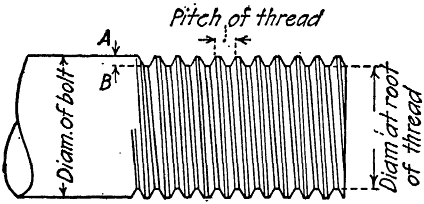


FIG. 28.

= diameter of bolt − 2 × 0.65 × the pitch.

The pitch $= \frac{1}{T}$ where T is the number of threads per in.

Therefore diameter at root of thread for U. S. Std.

$$= \text{diameter of bolt} - \frac{1.30}{T}$$

17. Construction of a Single, Square Thread, 2 Threads Per Inch. Fig. 29 Shows the Method of Construction.

18. Construction of an Acme Thread.—Square threads are not always easily cut, and there has been developed a standard, which is flat at top and bottom and has its sides inclined at an angle of 14½° to the center line of

the thread section. Fig. 30 shows the section for such a thread and the dimensions of the threads for a pitch of 1 in.

The angle of 29° or the total angle formed by the sides

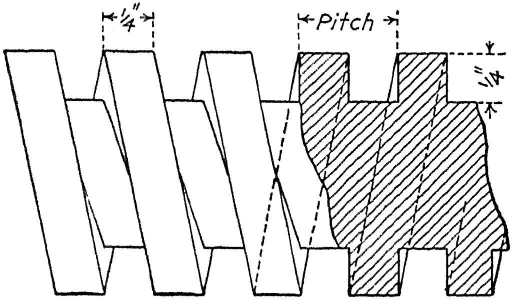


Fig. 29.

of an Acme thread as in Fig. 31 may be drawn with the use of a protractor or as shown in the figure which is drawn as follows:

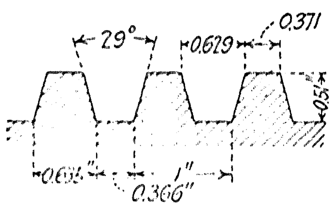


Fig. 30.

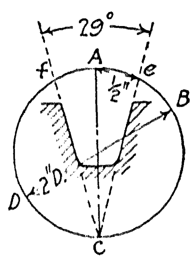


Fig. 31.

A circle $ABCD$ is drawn 2 in. in diam. and also a vertical diam. AC. From A as a center and with a radius of $\frac{1}{2}$ in., arcs are drawn cutting the circle at points e and f. Connecting these points with point C by the straight

lines *eC* and *fC* gives the inclination for the sides of the threads. The angle *eCf* is 29°.

In a "Worm" thread as in Fig. 32 the angle is the same as for the Acme thread, that is, 29°, but the thread is about one-third (1/3) deeper than the Acme.

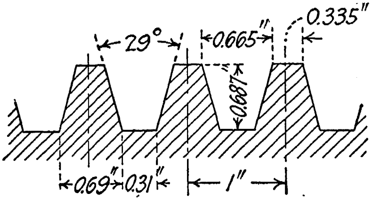


FIG. 32.

19. Miscellaneous Rules.—The width across flats of U. S. Std. hex. nuts equals $1\frac{1}{2}$ × Diameter of bolt + 1/8 in. See Fig. 33. The width across corners of U. S. Std. hex. nuts equals 1.155 × width across flats. The

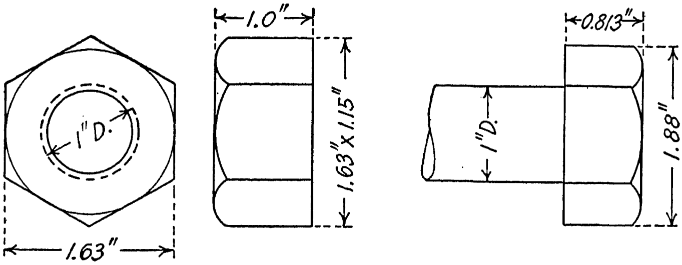


FIG. 33.

thickness of U. S. Std. hex. nuts (rough) is the same as diameter of bolt. Width across corners and flats of bolt heads is the same as for hex. nuts. The thickness of bolt heads equals half the width across flats of bolt head or 3/4 × diameter of bolt + 1/16 in.

TABLE FOR U.S. STD. THREADS

Diam. of screw	Threads to inch	Pitch	Depth of thread	Diam. at root of thread	Width of flat
1/4	20	.0500	.0325	.185	.0063
5/16	18	.0556	.0361	.2403	.0069
3/8	16	.0625	.0405	.2936	.0078
7/16	14	.0714	.0461	.3447	.0089
1/2	13	.0769	.0499	.4001	.0096
9/16	12	.0833	.0541	.4542	.0104
5/8	11	.0909	.0591	.5069	.0114
3/4	10	.1000	.0649	.6201	.0125
7/8	9	.1111	.0721	.7307	.0139
1	8	.1250	.0812	.8376	.0156
1 1/8	7	.1429	.0928	.9394	.0179
1 1/4	7	.1429	.0928	1.0644	.0179
1 3/8	6	.1667	.1082	1.1585	.0208
1 1/2	6	.1667	.1082	1.2835	.0208
1 5/8	5 1/2	.1818	.1181	1.3888	.0227
1 3/4	5	.2000	.1299	1.4902	.0250
1 7/8	5	.2000	.1299	1.6152	.0250
2	4 1/2	.2222	.1444	1.7113	.0278
2 1/4	4 1/2	.2222	.1444	1.9613	.0278
2 1/2	4	.2500	.1624	2.1752	.0313
2 3/4	4	.2500	.1624	2.4252	.0313
3	3 1/2	.2857	.1856	2.6288	.0357
3 1/4	3 1/2	.2857	.1856	2.8788	.0357
3 1/2	3 1/4	.3077	.1998	3.1003	.0385
3 3/4	3	.3333	.2165	3.3170	.0417
4	3	.3333	.2165	3.5670	.0417
4 1/4	2 7/8	.3478	.2259	3.7982	.0435
4 1/2	2 3/4	.3636	.2362	4.0276	.0455
4 3/4	2 5/8	.3810	.2474	4.2551	.0476
5	2 1/2	.4000	.2598	4.4804	.0500
5 1/4	2 1/2	.4000	.2598	4.7304	.0500
5 1/2	2 3/8	.4210	.2735	4.9530	.0526
5 3/4	2 3/8	.4210	.2735	5.2030	.0526
6	2 1/4	.4444	.2882	5.4226	.0556

TABLD FOR SHARP V THREADS

Diam. of screw	No. threads per inch	Pitch	Depth of thread	Diam. at root of thread
1/4	20	.0500	.0433	.1634
5/16	18	.0556	.0481	.2163
3/8	16	.0625	.0541	.2667
7/16	14	.0714	.0618	.3140
1/2	12	.0833	.0722	.3557
9/16	12	.0833	.0722	.4182
5/8	11	.0909	.0787	.4676
11/16	11	.0909	.0787	.5301
3/4	10	.1000	.0866	.5768
13/16	10	.1000	.0866	.6393
7/8	9	.1111	.0962	.6826
15/16	9	.1111	.0962	.7451
1	8	.1250	.1083	.7835
1 1/8	7	.1429	.1237	.8776
1 1/4	7	.1429	.1237	1.0026
1 3/8	6	.1667	.1443	1.0864
1 1/2	6	.1667	.1443	1.2114
1 5/8	5	.2000	.1733	1.2784
1 3/4	5	.2000	.1733	1.4034
1 7/8	4 1/2	.2222	.1924	1.4902
2	4 1/2	.2222	.1924	1.6152
2 1/8	4 1/2	.2222	.1924	1.7402
2 1/4	4 1/2	.2222	.1924	1.8652
2 3/8	4 1/2	.2222	.1924	1.9902
2 1/2	4	.2500	.2165	2.0670
2 5/8	4	.2500	.2165	2.1920
2 3/4	4	.2500	.2165	2.3170
2 7/8	4	.2500	.2165	2.4420
33	3 1/2	.2857	.2474	2.5052
1/8	3 1/2	.2857	.2474	2.6301
3 1/4	3 1/2	.2857	.2474	2.7551
3 3/8	3 1/4	.3077	.2666	2.8418
3 1/2	3 1/4	.3077	.2666	2.9668
3 5/8	3 1/4	.3077	.2666	3.0918
3 3/4	3	.3333	.2886	3.1727
3 7/8	3	.3333	.2886	3.2977
4	3	.3333	.2886	3.4227

CHAPTER V

CALCULATION OF LEVERS

20. Definitions and Problems.—A lever is a rod, bar or beam of any shape, either straight or curved, which is free to turn about a fixed point called a "fulcrum." The fulcrum may be any form of support, prop or bearing.

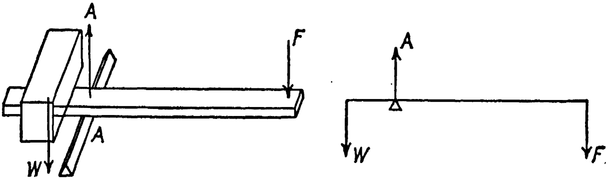


FIG. 34.

Levers are divided into three classes according to the positions of the loads, pressures, or forces acting on the lever with respect to the position of the fulcrum. In

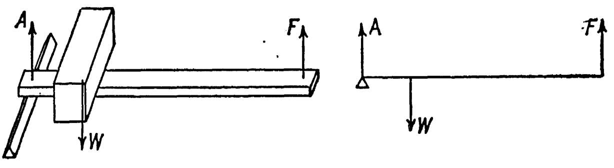


FIG. 35.

the first class, Fig. 34 the fulcrum A is between the weight (W) and the force (F) applied to lift the weight.

In the second-class lever Fig. 35 the weight W is between the fulcrum A and the force F.

In the third-class lever the force F is between the fulcrum A and the weight W, as in Fig. 36.

In working out lever problems we find what are called the "moments" of the forces or pressures or weights acting on the lever. In Fig. 37 which shows a lever of the first class the force F, 4 ft. from the fulcrum A,

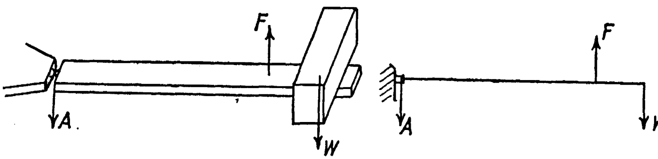


Fig. 36.

raises the weight W of 100 lb. located 2 ft. from A. Multiplying F by 4 lb., its distance from A gives the *moment* of F about the "fulcrum" A. Similarly $W \times 2$ ft. is the moment of W about the "fulcrum" A. To find what force is necessary at F to raise a weight of 100 lb. at W, we multiply $F \times 4$ ft. giving the moment of

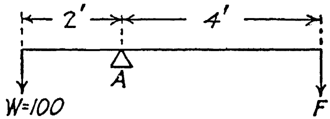


Fig. 37.

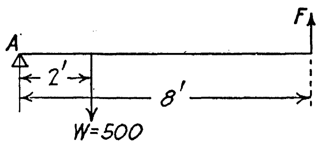


Fig. 38.

F about A and place this product equal to 100×2 which is the moment of the weight of 100 lb. about A. Then

$$100 \times 2 = F \times 4$$

therefore

$$200 = 4F$$

and

$$F = \frac{200}{4} \text{ or } 50 \text{ lb.}$$

Therefore 50 lb. pull at F will lift 100 lb. at W neglecting friction and the weight of the lever itself. The rule in any case is, find the moment of the force about the fulcrum and place this equal to the moment of the weight about the fulcrum and solve for the quantity desired.

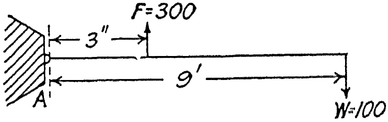


FIG. 39.

In Fig. 38 to find a force F necessary to lift a weight of 500 lb. at the distance given we have $500 \times 2 = F \times 8$ therefore

$$1000 = 8F$$

and

$$F = 125 \text{ lb.}$$

In Fig. 39 which represents a lever of the third class a force of 300 lb. at F will lift only 100 lb. at W, since taking the "moments" about the fulcrum A we have $300 \times 3 = W \times 9$.

Therefore

$$900 = 9W$$

and

$$W = 100 \text{ lb.}$$

PROBLEMS

What class of lever does Fig. 40 show and why? From the rules already given and using Fig. 40, fill out the following table,

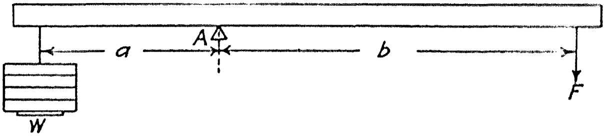


FIG. 40.

giving also sample calculations to show how you obtained your results. Neglect the weight of the lever itself.

No.	*W*, lb.	*F*, lb.	*a*	*b*
1	30		1 ft.	3 ft.
2		50	2 ft.	8 ft.
3	60	30		6 ft.
4	80	10	2 ft.	
5	18		3½ ft.	4¾ ft.
6		40	8 in.	2 ft.

What class of lever does Fig. 41 show and why? Using Fig. 41 fill out the following table, giving also sample calculations to

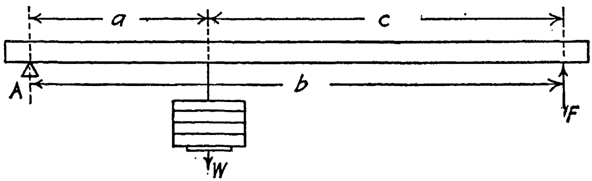


Fig. 41.

show how you obtained your results. Neglect the weight of the lever itself.

No.	*F* lb.	*W* lb.	*a*	*b*	*c*
7	50	200	8 ft.		
8	30		6 ft.		2 ft.
9		10	1 ft.	5 ft.	
10	30	60		4 ft.	
11		20	6½ ft.		2 ft.
12	27			39 in.	27 in.

What class of lever does Fig. 42 show, and why? Using Fig. 42 fill out the following table, giving also sample calculations to show how you obtained your results. Neglect the weight of the lever itself.

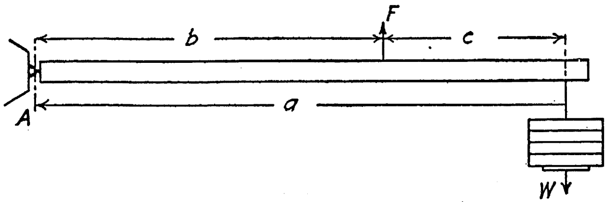


FIG. 42.

No.	*W*, lb.	*F*, lb.	*a*	*b*	*c*
13	50	200	8 ft.		
14	30		6 ft.		2 ft.
15		10	10 ft.	5 ft.	
16	30	60		4 ft.	
17		20	6½ ft.		2 ft.
18	27			39 in.	27 in.

19. As shown in Fig. 43, what downward force at *F* is necessary to start the stone if the resistance offered is 300 lb. If the stone

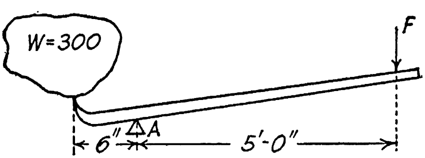


FIG. 43.

could not be moved with the bar as shown how could a greater force or pressure be produced on the stone, using the same bar? What class of lever does this represent?

20. In Fig. 44 the lever arms are bent at right angles to each other, allowing the effect of the weight *W* to be produced in a horizontal direction at *F*. With the lever arms as shown, what weight is necessary at *W* to produce a pull of 100 lb. at *F*?

21. In Fig. 45, showing a safety valve for a stationary boiler,

at what pressure will the boiler "blow off" with the size of valve and lever arms as shown? What class of lever does this show?

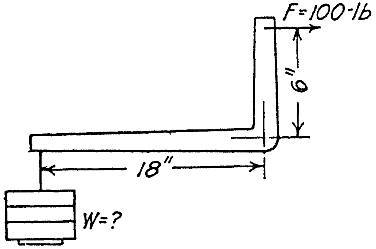


Fig. 44.

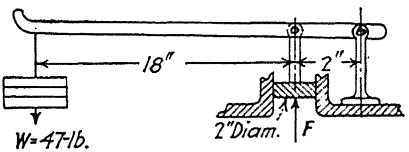


Fig. 45.

21. Calculation for Levers Taking into Account the Weight of the Lever Itself in Each Case.—If the lever shown in Fig. 46 weighs .5 lb. per in. of length the portion to the left of the fulcrum weighs 8 × .5 or 4 lb.

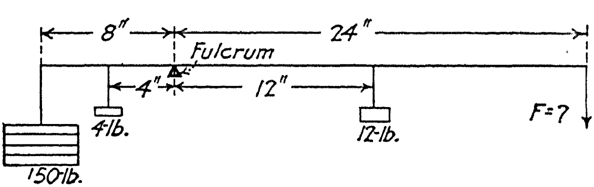


Fig. 46.

The portion to the right of the fulcrum weighs 24 × .5 or 12 lb. These weights should be taken at the *center of gravity* of the respective parts of the lever.

For example, the entire weight of the 8-in. length is

considered as concentrated at a point 4 in. from the fulcrum, that is, at the center of gravity of the 8-in. length. Likewise the 12 lb. is considered as concentrated at a distance of 12 in. from and at the right of the fulcrum. We now have four forces to deal with and our equation becomes:

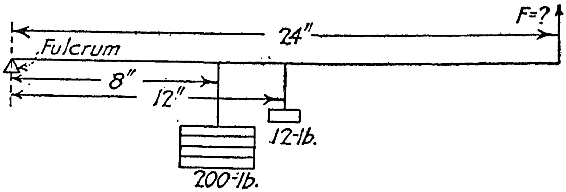


FIG. 47.

$$\begin{aligned} 150 \times 8 + 4 \times 4 &= 12 \times 12 + F \times 24 \\ 1200 + 16 &= 144 + 24F \\ 1200 + 16 - 144 &= 24F \\ 1072 &= 24F \\ 44.7 \text{ lb.} &= F \end{aligned}$$

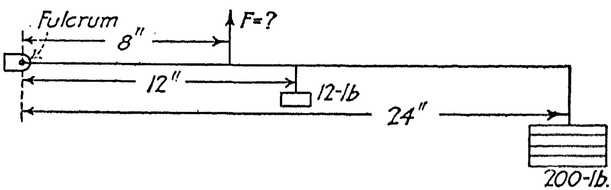


FIG. 48.

In Fig. 47, the lever weighs .5 lb. per in. of length or 12 lb. total weight. This acts at a distance of 12 in. from the fulcrum, that is, at the *center of gravity* of the whole lever. The equation then becomes

$$\begin{aligned} 200 \times 8 + 12 \times 12 &= F \times 24 \\ 1744 &= 24F \\ 72.6 \text{ lb.} &= F \end{aligned}$$

In Fig. 48, the lever weighs 12 lb. and the equation becomes

$$12 \times 12 + 200 \times 24 = 8F$$
$$4944 = 8F$$
$$618 \text{ lb.} = F$$

PROBLEMS

22. In a lever of the first class a weight of 250 lb. 6 in. from the fulcrum can be balanced by what force applied $1\frac{1}{2}$ ft. from the fulcrum?

23. Work out the above problem if the lever itself weighs 4 lb. per ft. of length.

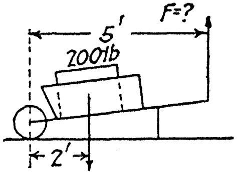


FIG. 49.

24. Find the force F required to lift the weight of 200 lb. on the wheelbarrow as shown in Fig. 49.

25. From Fig. 50 find the force which can be exerted at F through the air cylinder and system of levers shown.

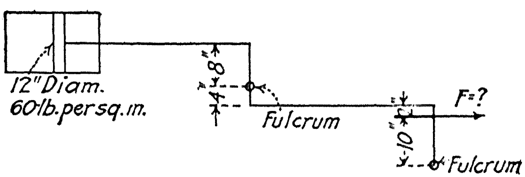


FIG. 50.

26. From Fig. 51, showing a safety valve for a stationary boiler, what must be the length of the arm "A" in order that the boiler will "blow off" at 150 lb. pressure per sq. in.? What class of lever does this sketch show?

27. From Fig. 52, showing a safety valve for a stationary boiler, what weight must be placed at W in order that the boiler will "blow off" at 150 lb. pressure per sq. in.?

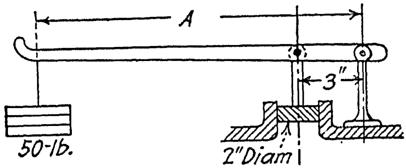


FIG. 51.

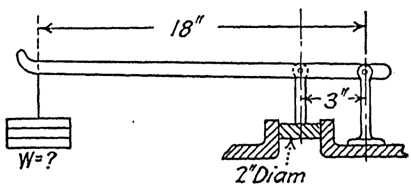


FIG. 52.

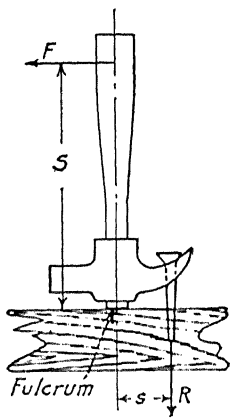


FIG. 53.

28. In using an ordinary claw hammer for pulling a nail as shown in Fig. 53, we are using a first-class lever. Suppose as shown

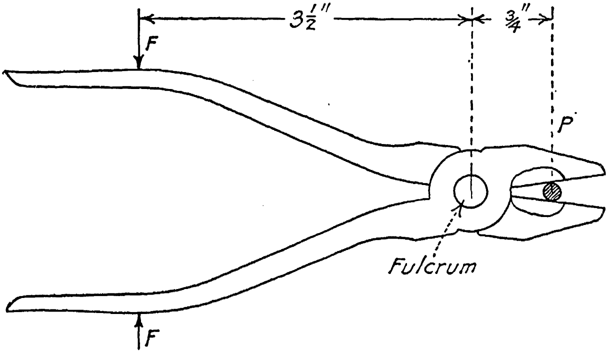


FIG. 54.

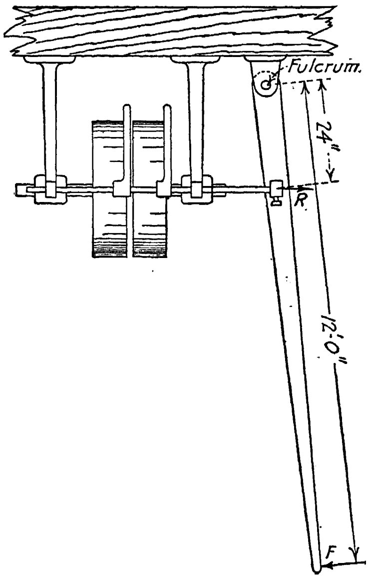


FIG 55.

a force of 40 lb. is exerted at "*F*," we find the resistance of the nail at the start by calculating a first-class lever, whose arms are bent up at right angles to each other. If the distance $s = 2$ in. and $S = 9$ in., calculate the resistance "*R*" of the nail at the start.

29. The pliers and wire cutters shown in Fig. 54 form a first-class lever. If the total force on the handle of the pliers is 10 lb. at a distance of $3\frac{1}{2}$ in. from the fulcrum, how much pressure is produced on the wire at "*P*" in cutting it? The wire is $\frac{3}{4}$ in. from the fulcrum.

30. Fig. 55 shows a "belt shifter." If the resistance at "*R*," required to be overcome in shifting the belt is 35 lb., what force is required at "*F*" to move the belt with the lever arms shown?

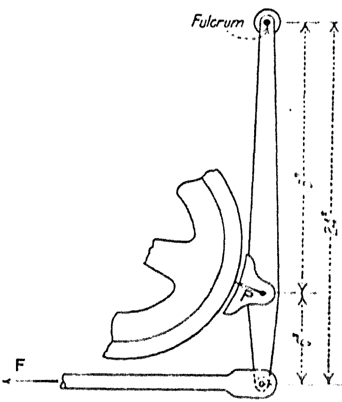


Fig. 56.

31. If in shifting the belt it is necessary to move the shifter rod 4 in. how far does the end of the long arm move, that is, the end on which the force "*F*" is exerted?

In Fig. 56 is shown a second-class brake lever with brake shoe and brake rod. The motion of the brake shoe is to the motion of the pull rod as 18 : 24 with the lengths of lever arms as shown. For example if the

brake rod moves 4 in. the shoe movement is found as follows:

$$\frac{\text{Movement of brake shoe}}{\text{Movement of pull rod}} = \frac{18}{24} \text{ or } \frac{\text{movement of brake shoe}}{4} = \frac{18}{24}$$

That is, movement of brake shoe $= 4 \times \frac{18}{24} = 3$ in.

Also if the braking force through the brake shoe is called "P" we have the proportion $\frac{P}{F} = \frac{24}{18}$ from which $P = \frac{F \times 24}{18}$.

Example.—If the pull on the rod is 4800 lb., that is, if $F = 4800$ lb. then $P = \frac{4800 \times 24}{18}$ or 6400 lb.

32. In the brake lever shown in Fig. 56, if the distance from the fulcrum to brake shoe is 24 in. and the whole effective length of lever is 30 in. what is the movement of brake shoe compared with that of the brake rod? What pull is necessary to produce a pressure of 8000 lb. against the wheel?

33. Find the length of lever arms for a brake lever as shown in Fig. 56 such that a pull of 800 lb. on the brake rod will produce a pressure of 4000 lb. of the brake shoe against the wheel. In this case what is the ratio of movement of brake shoe and brake rod?

Levers used to Increase or Decrease Motion.—Levers are often used to increase motion which of itself is too small to read easily as in the case of an indicator as shown in Fig. 57. With this instrument the piece to be measured is put in at "G." The instrument is "calibrated" or compared with a standard gauge to find the reading on the scale "S" for a given gauge. When the piece is put in at "G" any movement of the pointer above or below the gauge point on "S" shows how much too large or too

small the piece is that is being measured. The reading of the scale for different pieces tested will depend on the ratio of the arms of the lever used. If the short arm is .5 in. and the long arm 10 in. as shown the ratio of the motion of the ends of the lever is as 1 : 20. If for

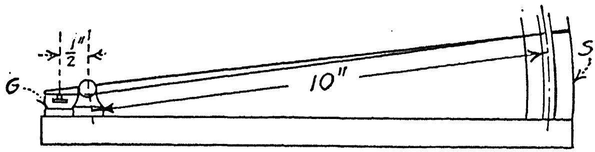


FIG. 57.

example in testing a piece the end of the long lever moves .2 in. beyond the standard gauge mark, the work being measured is one-twentieth of .2 or .01 in. from the standard.

PROBLEMS

34. In the indicator shown in Fig. 57 the pointer indicates on scale "*S*" a distance of .48 in. beyond the gauge mark. How much is the piece being measured out of gauge?

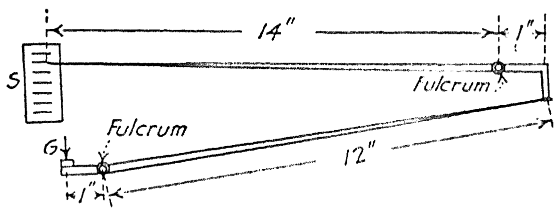


FIG. 58.

35. An indicator similar to that shown in Fig. 57 has lever arms of .75 in. and 18 in. If a piece being measured is .025 in. from standard gauge, how far will the pointer move on scale "*S*" from the standard gauge mark?

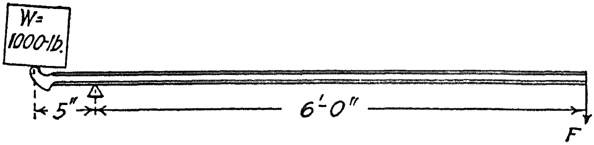


FIG. 59.

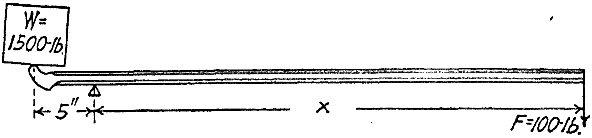


FIG. 60.

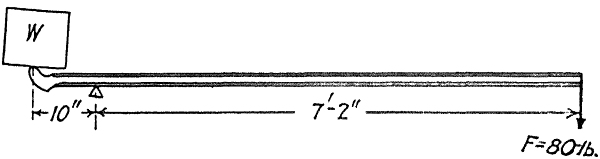


FIG. 61.

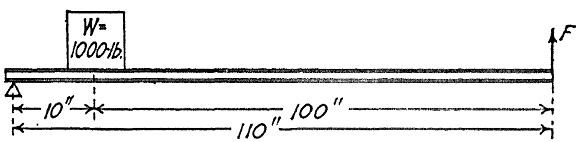


FIG. 62.

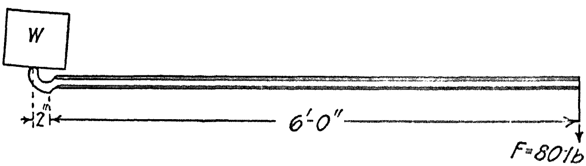


FIG. 63.

36. In the lever indicator system shown in Fig. 58, find the motion produced at "S" by a motion of .040 in. at G.

37. Show how you could redesign the levers so that a motion of .040 in. at G would produce at "S" twice the motion calculated in the problem above.

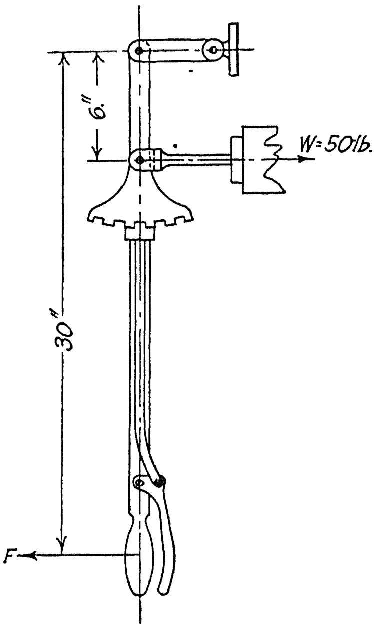


Fig. 64.

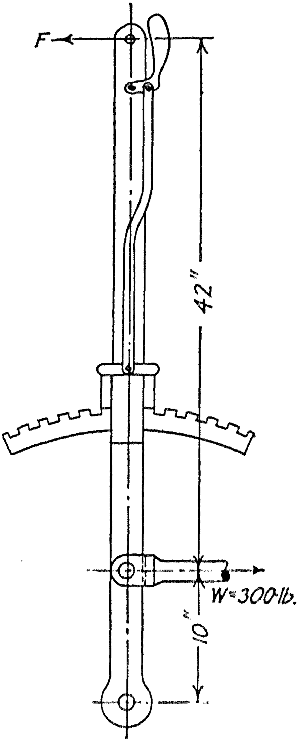
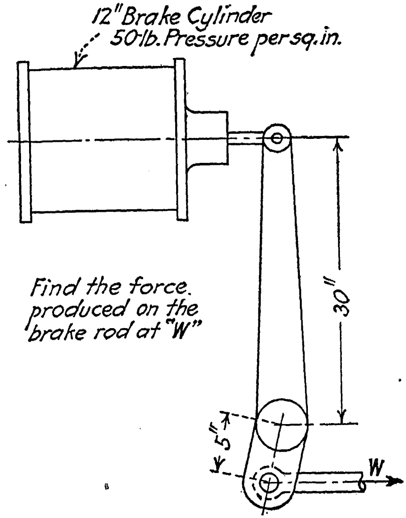


Fig. 65.

38. In Fig. 59, find the force F necessary to raise the weight W.

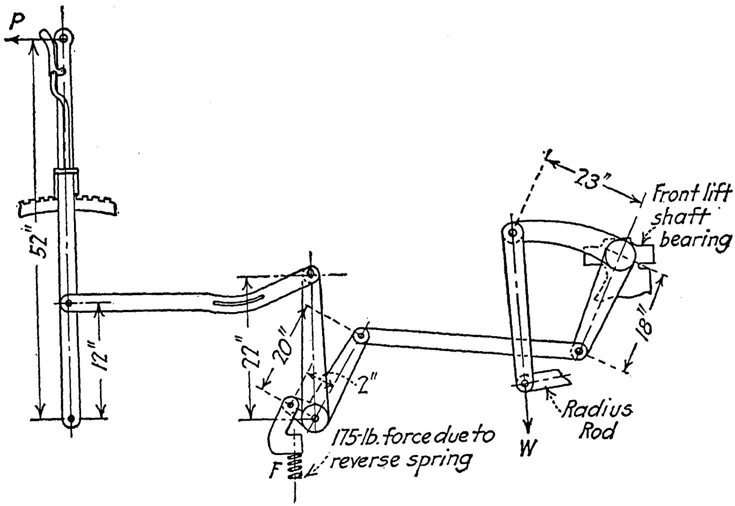
39. In Fig. 60, find the length of the lever arm X so that the force of 100 lb. will balance the load W of 1500 lb.

40. In Fig. 61, find the weight W that can be raised by the force of 80 lb.



12" Brake Cylinder
50-lb. Pressure per sq. in.
Find the force produced on the brake rod at "W"
30"
5"
W

Fig. 66.



P
52"
12"
22"
20"
2"
23"
Front lift shaft bearing
18"
Radius Rod
W
175-lb. force due to reverse spring
F

Fig. 67.

41. In Fig. 62, find the force F required to raise the weight W.

42. In Fig. 63, showing a pinch bar 6 ft. long with 2-in. nose, find the weight W that can be raised.

43. In Fig. 64, showing a throttle lever, find the force F required to move the throttle.

44. Fig. 65 shows a reverse lever. Find the force F required to reverse the engine.

45. Fig. 66 shows a brake cylinder. Find the force produced on the brake rod at W by the pressure of 50 lb. per sq. in. in the cylinder.

46. With a load at W, Fig. 67, of 400 lb. due to lifting the link blocks, radius rod ends and moving the valves, what will be the force required at P to move the reverse lever backward taking into account the action of the reverse spring, but neglecting the friction of the reversing gear?

CHAPTER VI

PULLEYS (BLOCK AND TACKLE)

22. Arrangement of Pulley Systems.—Pulleys or blocks (Fig. 68) are used to change the direction of application of a force, as well as to reduce the value of the force necessary to lift a given weight or overcome a given resistance. Pulleys are in fact *rotating levers*.

With a single pulley as in Fig. 68, the force F applied to raise the weight W, can be exerted *downward* instead of upward. This makes it easier to raise the weight. In this case the force required at F is equal to the weight lifted at W, neglecting friction.

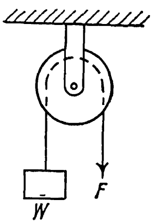


FIG. 68.

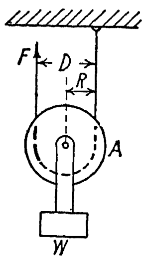


FIG. 69.

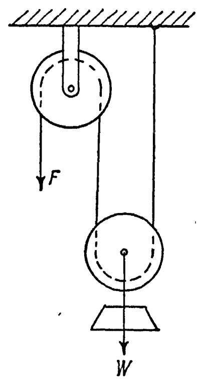


FIG. 70.

This arrangement forms a *lever* of the first class. Compare this with a first-class lever.

In the case of a *single movable* pulley, as in Fig. 69, the force necessary at F is one-half the weight lifted at

W, neglecting the effect of friction. This arrangement forms a *lever* of the *second* class and we have the weight $W \times$ radius of pulley $= F \times$ diameter of pulley. Therefore to lift 100 lb. at W requires a force of 50 lb. at F. Compare this figure with a second-class lever.

Fig. 70 shows a *fixed* and a *movable* pulley, or a combination of the arrangements shown in Figs. 68 and 69. The force required at F to lift a weight W is

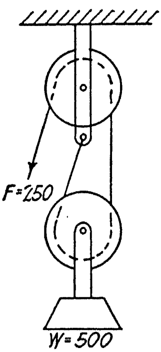


FIG. 71.

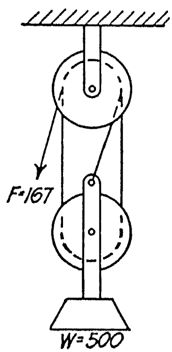


FIG. 72.

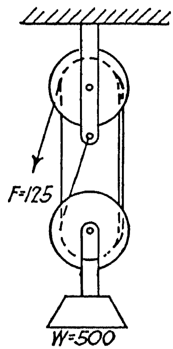


FIG. 73.

equal to one-half of the weight neglecting friction of the rope and pulleys. When the force F is exerted through a distance of 2 ft., the weight is lifted 1 ft. A force of 50 lb. at F will raise a weight of 100 lb. at W. The friction of the rope on the pulleys and the pulleys on their bearings is usually small compared to the weights ordinarily raised, and this friction is reduced by the use of grease or oil on the bearings.

In the following arrangements of blocks, Figs. 71, 72, and 73, the force required to raise a given weight

or vice versa is found as follows: *Rule.*—The force in pounds multiplied by the distance in feet through which it moves, equals the weight in pounds lifted, multiplied by the distance through which it moves, and another rule which also applies for any number of pulleys in either block is: Divide the weight to be lifted by the number of *single* strands or lines of rope extending down to the movable pulley, to find the force F. The distance through which the weight moves equals the distance through which the force F moves divided by the number of single lines of rope extending down to the movable block. In Fig. 71 a force of 250 lb. is necessary to lift a weight of 500 lb., neglecting friction. For each of the arrangements shown the force necessary to raise a weight of 500 lb. is given.

PROBLEMS

1. With a single movable pulley as in Fig. 69, what force is necessary at F to raise a weight of 300 lb.? What advantage has the arrangement shown in Fig. 70 over this one using only the single movable pulley?

2. In Fig. 71 what force at F is necessary to lift 100 lb. at W? What weight at W will just balance a downward pull at F of 230 lb.?

3. With arrangement as shown in Figs. 72 and 73, does the size of pulleys have anything to do with the force required to lift a given weight?

4. In Fig. 72, how much force is necessary at F to raise a weight of 800 lb. at W?

5. In Fig. 72, what weight at W will just balance a pull at F of 200 lb. if the lower block and hook together weigh 20 lb.?

6. In Fig. 73, what weight could a man weighing 150 lb. raise by hanging on the rope at the end F?

If the weight was raised 1 ft., through how many feet does the man move?

7. What force at F will a weight of 1000 lb. at W just balance

with the arrangement of Fig. 73, if the lower block and hook together weigh 26 lb.?

The Differential Pulley.—For lifting heavy weights by hand a differential pulley is often used.

As shown in Fig. 74, this consists of two pulleys A and B rigidly fastened together and rotating as one about a fixed axis C. An endless chain passes over both pulleys, and one of the loops of the chain passes under and supports the lower or movable block D. The other loop of the chain hangs freely and is the loop upon which one pulls in raising a weight on the hook E. The rims of the pulleys are grooved and carry lugs to prevent the chain from slipping. If the radii of the large and small fixed pulleys are R and r respectively, "P" the pulling force exerted on the chain and "W" the weight lifted, the rule for the pulley is as follows, leaving out friction:

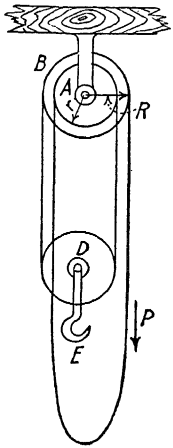


FIG. 74.

$$P \times R = W \times \tfrac{1}{2}(R - r)$$

or

$$\frac{W}{P} = \frac{R}{\tfrac{1}{2}(R - r)}$$

As used a differential pulley can actually lift only about 25 to 35 per cent. of the weight calculated for a given pull from the formula stated above. This means therefore that to lift a given weight requires actually from three to four times the pull calculated by the formula which does not take account of friction.

Example.—What force is necessary to lift a weight of 800 lb. with a differential hoist having pulleys 10 in. and $8\frac{1}{2}$ in. in diam. and an efficiency of 25 per cent.?

Solution.—$P \times R = W \times \frac{1}{2}(R - r)$

$$\text{that is } P = \frac{W \times \frac{1}{2}(R - r)}{R}$$

$$\text{or } P = \frac{800 \times \frac{1}{2}(5 - 4\frac{1}{4})}{5}$$

$$= \frac{800 \times \frac{1}{2} \times \frac{3}{4}}{5}$$

or $$\frac{800 \times \frac{3}{8}}{5} \text{ or } 60 \text{ lb.}$$

Since the efficiency is only 25 per cent. the actual pull required is $\frac{60}{.25}$ or 240 lb.

PROBLEMS

8. From the rule for a differential pulley find the force necessary to lift a tank weighing 280 lb. The diameters of the pulleys being 12 in. and 10 in. and the efficiency 30 per cent.

9. In a differential hoist with pulleys 11 in. and 9½ in. and an efficiency of 53 per cent., find what weight a 150-lb. man can raise by putting his own weight on the chain.

CHAPTER VII

THE INCLINED PLANE AND WEDGE. THE SCREW JACK

23. The Inclined Plane and Wedge.—A block whose sloping surface is less than 90° may be considered an "inclined plane." In Fig. 75 which shows such an inclined plane, AB is the "base," BC the "height" and AC the "face" or "sloping surface." With the use of the inclined plane a given resistance can be overcome with a smaller force than if the plane were not used. For ex-

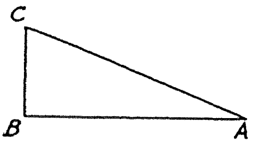


FIG. 75.

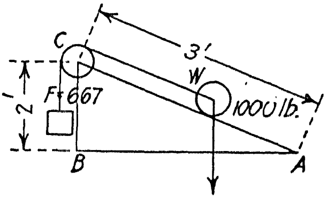


FIG. 76.

ample, in Fig. 76, suppose we wished to raise a weight of 1000 lb. through the vertical distance $BC = 2$ ft. If this weight were raised vertically and without the use of the plane the force of 1000 lb. would have to be exerted through the distance BC. If, however, the inclined plane is used and the weight is moved over its face AC, a force of only $\frac{2}{3}$ of 1000 lb. or 667 lb. is necessary, although this force is exerted through a distance AC which is greater than distance BC. With the

inclined plane, we, therefore, require a smaller force which must be exerted through a greater distance to do a certain amount of work.

Letting F represent the force required to raise a given weight on the inclined plane, and W the weight to be raised, we have the proportion:

$$\frac{F}{W} = \frac{\text{height of plane}}{\text{length of sloping surface of plane}}$$

or for Fig. 76

$$\frac{F}{W} = \frac{2}{3} \text{ and } \frac{F}{1000} = \frac{2}{3}$$

therefore $F = \frac{2}{3} \times 1000$ or 667 lb.

Stated in words the rule for the "Inclined Plane" is:

The force required to raise a given weight over the plane is to the weight to be raised as the *height* of the plane is to the *length* of the sloping surface.

To find the force required, multiply the weight on the plane by the height of the plane and divide by the length of the face of the plane.

To find the weight which a given force will raise, multiply the force by the length of the face of the plane and divide by the height of the plane.

These rules do not include the effect of *friction* on the plane.

As shown in Fig. 77, a "wedge" consists of two inclined planes with their bases together. The force F used to drive the wedge acts parallel to the center line of the wedge.

The principle of the wedge is the same as that for the inclined plane. In this case the wedge in being driven in is the same as if the weight being raised moved along a stationary inclined plane. Leaving out the effect of

friction on the sides of the wedge, the work done in driving the wedge equals the force exerted, F times l or Fl. The work done in raising the weight equals the weight lifted "W" times the distance lifted "h" or Wh. Then leaving out friction

$$Fl = Wh \text{ or } \frac{F}{W} = \frac{h}{l} \text{ or } F = W \times \frac{h}{l}$$

Therefore the force required is only $\frac{h}{l}$ times the weight lifted.

Example.—In Fig. 78, a wedge 12 in. long has a taper

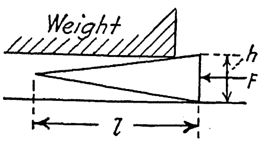


FIG. 77.

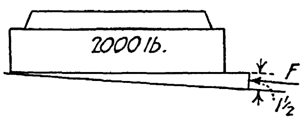


FIG. 78.

of 1½ in. per ft. and is used to raise a machine base weighing 1 ton. The force F required is found as follows:

$$F = \frac{W \times h}{l} = \frac{2000 \times 1.5}{12} = 250 \text{ lb.}$$

The "mechanical advantage" in using the wedge is therefore $\frac{2000}{250}$ or 8. That is, the force required is only ⅛ of the weight lifted.

PROBLEMS

1. A weight of 2½ tons is to be raised by a wedge having a taper of 1 in. per ft. What force is required to drive the wedge and what is the mechanical advantage leaving out the effect of friction?

2. To wedge up a weight with a wedge tapering 2 in. per ft. requires a force of 500 lb. Neglecting friction, find the weight raised.

24. The Screw Jack.—One of the most common applications of the principle of the inclined plane is in the ordinary screw thread, which is nothing but an inclined plane wound around a cylinder.

If we cut out a wedge-shaped piece from thick paper and wind it around a cylinder we will see that the sloping side forms a thread. The form of this thread as in Fig. 79 is called a "helix." All nut, bolt, jack, and screw threads are circular or spiral wedges.

Fig. 80 represents a screw

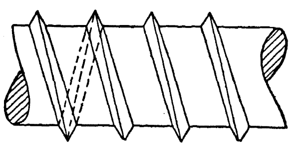


FIG. 79.

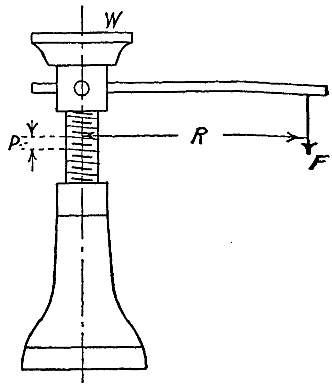


FIG. 80.

jack which is used to overcome a heavy pressure or raise a heavy weight at W by a much smaller force F applied at the handle. R represents the length of the handle and P the *pitch* of the screw, or the distance the screw advances in one complete turn.

Neglecting friction the following rule is used: The force F multiplied by the distance through which it moves in one complete turn is equal to the weight lifted times the distance through which it is lifted in the same time. In one complete turn the end of the handle describes a

circle of circumference $2\pi R$. This is the distance through which the force F is exerted.

Therefore from the rule above

$$F \times 2\pi R = W \times P$$

$$\pi = 3.14 \qquad \text{and } F = \frac{W \times P}{2\pi R}$$

Suppose in Fig. 80, $R = 18$ in., $P = \frac{1}{8}$ in. and the weight to be lifted is 50 tons or 100,000 lb., the force in lb. required at F is then found as follows:

$$F = \frac{W \times P}{2\pi R}$$

$$F = \frac{100{,}000 \times \frac{1}{8}}{2\pi 18} \text{ or}$$

$$F = \frac{100{,}000}{8} \times \frac{1}{2\pi 18}$$

Cancelling we get $F = \dfrac{3125}{\pi 9} = \dfrac{3125}{28.3} = 110$ lb. necessary.

This means then that neglecting friction 110 lb. at F will raise 100,000 lb. at W, but the weight lifted moves *much slower* than the force applied at F.

PROBLEMS

3. What weight can be held by a force of 100 lb. on a plane which rises 4 ft. in every 5 ft. measured horizontally?

4. The height of an inclined plane is 10 ft. and its length is 20 ft. What weight will a force of 25 lb. at F in Fig. 76 hold up on the plane?

5. What counterbalance would be necessary to hold an empty coal car weighing 500 lb. on an incline 20 ft. long and 8 ft. high?

6. A screw jack has a single thread 8 threads per in. and the handle is 2 ft. long. What force is necessary at F to raise a weight of 2500 lb., neglecting friction?

7. What effect has increasing the pitch P, on the force necessary to raise a given weight if the length R does not change?

8. It is desired to jack up a machine with a square base and weighing 5 tons by four jacks, one at each corner. If the jacks have 6 threads per in. and $1\frac{1}{2}$-ft. handles, and the weight is evenly distributed, what force is necessary at F?

9. What effect has increasing the length of handle on the force F necessary to raise a given weight if the same pitch is used?

10. A force of 50 lb. at F raises a weight of 3768 lb. at W with $R = 1\frac{1}{2}$ ft. What is the pitch of the screw?

CHAPTER VIII

GEARS, LATHE GEARING

25. Definitions.—A gear, or toothed wheel, when in operation, may be considered as a lever of the first class with the addition that it can be rotated continuously instead of rocking back and forth through a short distance. What we should learn here is the relation between the number of teeth, the diameter, and the speed of gears. Fig. 81 shows the ends of two shafts A and B connected by 2 gears of 24 and 48 teeth respectively. Notice that the larger gear will make only one-half turn while the smaller makes a complete turn. That is, the ratio of speeds of the larger to the smaller is as 1 is to 2, or expressed as a proportion.

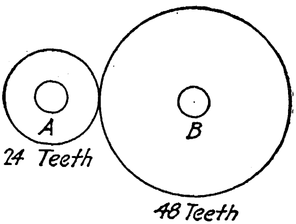


FIG. 81.

Speed of B: Speed of A $= 1:2$ and the ratio between the speeds and the number of teeth written as a proportion is

Speed of B: Speed of A $=$ Teeth of A : Teeth of B.

If in Fig. 81 gear A had 24 teeth and B 96 teeth, the smaller gear would make four turns while the larger makes 1, or,

Speed of A: Speed of B $= 96:24$, that is, the gear with the smaller number of teeth must turn the faster.

Of two gears running together that one is called the

driver which is nearer the source of power and the second gear which receives power from the driver is called the *follower*.

26. Gear Trains (Simple Gearing).—In the case of *gear trains* there may be several drivers and several followers. When the teeth on a gear turn in the same direction as the hands of a clock the motion is called *right handed* or "clockwise," and when in the opposite direction, *left handed*, or "counter-clockwise." In Fig. 82 suppose the three gears A, B, and C have 96, 24, and 96 teeth respectively.

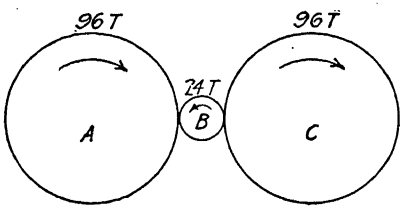


FIG. 82.

When gear A turns *once* right handed, gear B turns 4 *times* left handed and gear C turns *once* right handed. Hence gear B does not change the speed of C from what it would have been if geared directly to A, *but it changes its direction from* left handed to right handed.

The ratio of speeds of the first and last gears in a train of simple gears is not changed by putting any number of gears between them.

27. Gear Trains (Compound Gearing).—Fig. 83 shows "compound gears" in which there are two gears on the middle shaft. Gears B and D rotate at the same speed since they are keyed to the same shaft. If the gears have a number of teeth as shown by the numbers in the

figure and gear A makes 100 r.p.m. *right handed,* gear B turns 400 r.p.m. *left handed,* also gear D turns 400 r.p.m. *left handed* and gear C turns 1200 r.p.m. *right handed.*

28. Lathe Gears for Screw Cutting (Simple Gearing).—Modern lathes are equipped with a plate which gives the gears required for cutting different screw threads. To find the gears required in any case however it is merely necessary to understand the principle of a "simple" or a

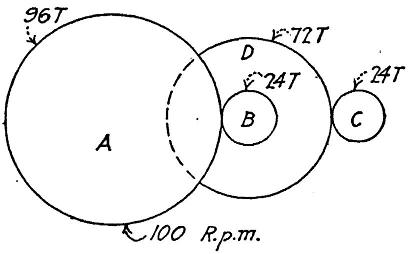


FIG. 83.

"compound" gear train depending on whether the lathe is simple or compound geared as explained in the following:

Fig. 84 shows a "simple geared" lathe with the names of important parts indicated. The lathe carriage carrying the cutting tool is moved by the lead screw which has usually 2, 4, 6, or 8 threads per in. If the lead screw has 4 threads per in., each complete revolution of this screw moves the carriage and hence the cutting tool ¼ in. along the work being threaded, and 4 turns moves the tool 1 in. along the work. If now we wish to cut 12 threads per in., the spindle must revolve 12 times while the lead screw revolves 4 times. That is,

$$\frac{\text{turns of spindle}}{\text{turns of lead screw}} = \frac{12}{4} \text{ or as } \frac{3}{1}$$

The gears on the lead screw and on the spindle must therefore have a ratio of teeth of 3 to 1, or

$$\frac{\text{teeth on lead screw gear}}{\text{teeth on spindle gear}} = \frac{3}{1}$$

since the lead screw turns the slower.

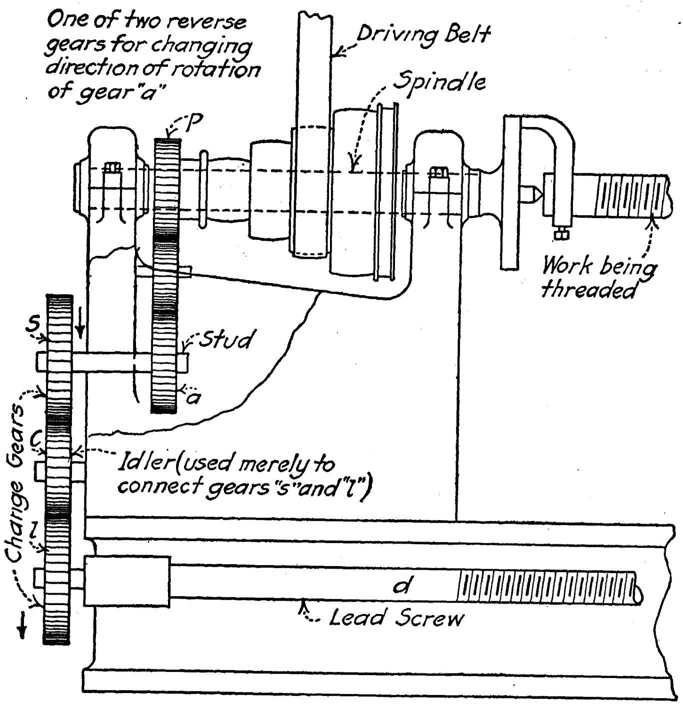


FIG. 84.

If the gear a on the stud has the same number of teeth as gear p on the spindle, we have the rule

$$\frac{\text{teeth of lead screw gear}}{\text{teeth of stud gear}} = \frac{3}{1}$$

The idler gear (c) is used to connect gears S and l, and its number of teeth does not influence the speed ratio of the spindle and lead screw. The same is true of the gear connecting the spindle with the stud.

The following is the rule: The number of threads to be cut per in. equals the product of the number of threads per in. in the lead screw with the number of teeth in each driven gear and divided by the product of the number of teeth in each driving gear. In the figure shown, if the lead screw has 4 threads per in., gear p on the spindle 24 teeth, gear a on the stud 48 teeth, gear S 36 teeth, and gear l 72 teeth, the threads cut per in. equal $\frac{4 \times 48 \times 72}{24 \times 36}$ or 16.

Using the letters on the gears to indicate their number of teeth we have the rule, if n = the number of threads cut per in. and

d = the number of threads per in. of lead screw

$$n = \frac{d \times a \times l}{p \times s}$$

For another example:

if	d =	threads per in. of lead screw	=	2
	a =	teeth in gear on stud	=	48
	p =	teeth in gear on spindle	=	24
	s =	teeth in outside gear on stud	=	24
	l =	teeth in lead screw gear	=	72

Then $n = \frac{2 \times 48 \times 72}{24 \times 24} = 12$ threads per in.

Suppose on the other hand we wished to find gears "s" and "l" to cut 8 threads per in. with a lead screw

having 6 threads per in. we have from the rule,

$$n = \frac{d \times a \times l}{p \times s}, \text{ and } \frac{l}{s} = \frac{n \times p}{d \times a}, \text{ from which}$$

$$\frac{l}{s} = \frac{8 \times p}{6 \times a} = \frac{8}{6} \times \frac{p}{a}$$

to find the ratio $\frac{l}{s}$ we must choose gears p and a. If we take p as 36 teeth and a 24 teeth, we have

$$\frac{l}{s} = \frac{8}{6} \times \frac{36}{24} = \frac{2}{1}$$

that is gears l and s must have a ratio of 2 : 1, hence we can put a 48-tooth gear on the lead screw and a 24-tooth gear as the outside gear on the stud and 8 threads per in. will be cut.

29. Lathe Gears for Screw Cutting (Compound Gearing).—In the compound geared lathe, as shown in Fig. 85, there are two change gears of different sizes on the spindle between the gear "*s*" on the stud and the gear "*l*" on the lead screw. The rule to be used in this case is as follows:

The threads to be cut per in. equal the product of the threads per in. of the lead screw and the teeth in each of the driven gears, divided by the product of the number of teeth in each of the driving gears. Representing the number of teeth of the gears by the letters on the gears we have, if "*n*" is the threads cut per in. and "*d*" the number of threads per in. of the lead screw.

$$n = \frac{d \times a \times l \times e}{p \times s \times f}$$

For example, suppose the lead screw has 5 threads per in. and the gears have a number of teeth as follows: $a = 28$, $p = 28$, $l = 72$, $f = 36$, $s = 24$, $e = 24$, the number of threads cut per in. according to the rule becomes

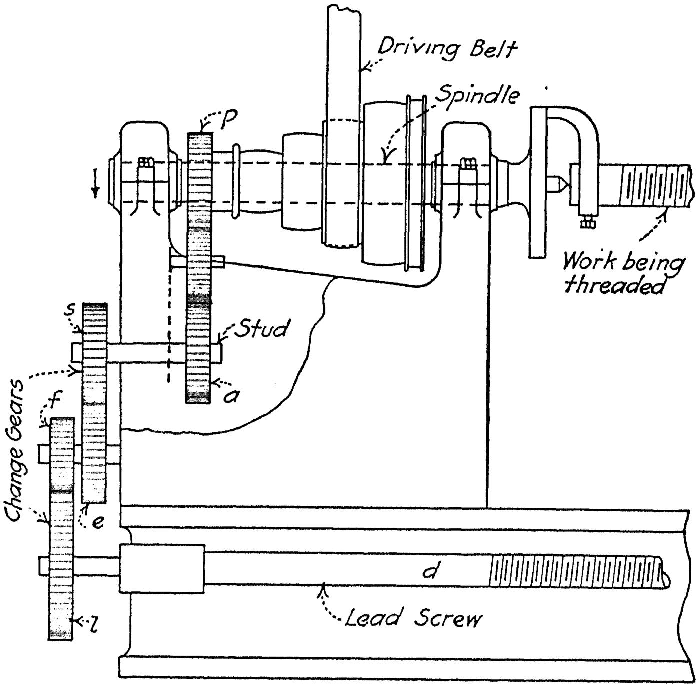


Fig. 85.

$$n = \frac{d \times a \times l \times e}{p \times s \times f} = \frac{5 \times 28 \times 72 \times 24}{28 \times 24 \times 36}, \text{ or } 10.$$

Working the other way round, if we wish to find the gears to use to cut a given thread we proceed as follows:

If $d = 5$ and the threads cut per in. are to be 14, we have

$$14 = \frac{5 \times a \times l \times e}{p \times s \times f}, \quad \text{from this}$$

$$\frac{14}{5} = \frac{a \times l \times e}{p \times s \times f}$$

The ratio of the gears must therefore be as 14 is to 5.

Suppose by trial we take $e = 28$ and $s = 30$ teeth, we then have

$$\frac{14}{5} = \frac{a \times l \times 28}{p \times 30 \times f} \quad \text{or} \quad \frac{14 \times 30}{5 \times 28} = \frac{a \times l}{p \times f}$$

that is $\dfrac{a \times l}{p \times f} = \dfrac{3}{1}$, the remaining ratio.

To provide this ratio we may take $f = 24$, $l = 72$, and a and p even gears as, for example, 36 teeth each.

We then have

$$\frac{a \times l}{p \times f} = \frac{36 \times 72}{36 \times 24} \quad \text{or} \quad \frac{3}{1}$$

PROBLEMS

1. If 8 threads are to be cut per in. and the lead screw has 6 threads per in., what must be the ratio of revolutions for the spindle and lead screw?

2. In a simple geared lathe calculate the size of change gears to use to cut 14 threads per in. if the lead screw has 5 threads per in.

3. In a lathe the ratio of revolutions of lead screw and spindle is $\frac{1}{3}$. If the pitch of the thread on the lead screw is $\frac{1}{4}$ in., how many threads are cut for 16 revolutions of the lead screw?

4. In a compound geared lathe calculate the size of change gears to use to cut 18 threads per in. if the lead screw has 4 threads per in.

In working out the following problems think first of the relative sizes of the gears, which is the driver and which the follower, and remember that of two gears the larger must turn slower than the smaller.

The diameters of gears are *proportional* directly to their number of teeth and *inversely* proportional to their speeds.

That is, if two gears A and B have pitch circles of diameters 12 and 24 in., respectively, and gear A has 48 teeth, gear B must have 96 teeth and if the speed of A is 100 revolutions per minute (r.p.m.) the speed of B is 50 r.p.m.

PROBLEMS

5. If two gears A and B have 24 and 72 teeth respectively, and gear A turns right handed, in which direction will gear B turn? While gear A turns three times, how many times will B turn?

6. If two shafts are to be connected by gears of 48 and 36 teeth, and the faster makes 900 r.p.m. what speed does the other make?

7. Two gears have one 120 and the other 90 teeth. What is the ratio of the diameters of their pitch circles and of their speeds?

8. Two gears have pitch circles 76 in. and 19 in. in diam. What is the ratio of their speeds?

9. It is desired to run a shaft at 200 r.p.m. from another running at 500 r.p.m., what gear must be used on the second shaft if the gear on the first shaft has 150 teeth?

10. If 7 gears work in a train in what direction will the last one turn if the first turns left handed? How could you change the direction of rotation of the last gear?

11. A train of gears is made up of 4 gears with the number of teeth as follows: 72, 48, 32, and 24. If the first gear makes 20 r.p.m. right handed, in what direction and at what speed will the last gear move?

12. A train of gears is made up of 5 gears with the number of teeth as follows: 48, 36, 24, 60, and 48. If the first gear makes 30 r.p.m. left handed, in what direction and at what speed will the last gear move?

13. A main shaft runs at 150 r.p.m., right handed, and carries a 96-tooth gear which drives a 24-tooth gear on a second shaft. A 72-tooth gear on the second shaft drives a 24-tooth gear on a third shaft. Find the speed of each shaft and make a sketch showing the arrangement of gears and directions of rotation produced.

14. A simple gear train has four gears with the number of teeth as follows: 96, 24, 48, and 60. If the last gear is on a shaft making 500 r.p.m. and turns right handed, find the speed and direction of rotation of the first gear.

15. According to Fig. 86, in what direction will gear 2 turn? Find the speed ratio of the gears.

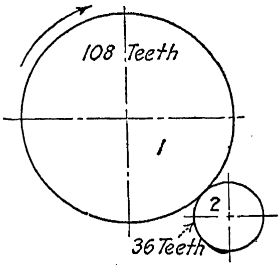


FIG. 86.

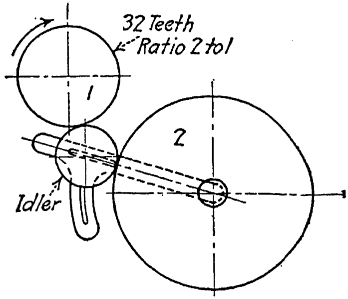


FIG. 87.

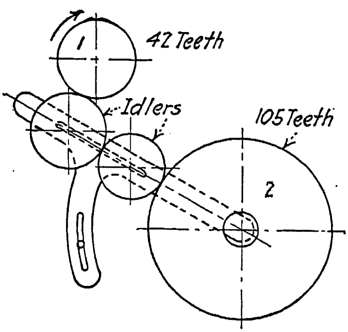


FIG. 88.

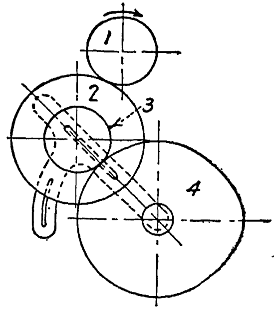


FIG. 89.

16. What is the direction of rotation of gear 2 according to Fig. 87? Find the number of teeth in gear 2.

17. Fig. 88 shows a train of simple gears. In what direction will gear 2 turn? Find the speed ratio of gears 1 and 2.

18. Fig. 89 shows a train of compound gears. In what direction will gear 4 turn.

CHAPTER IX

BELTS AND PULLEYS. EFFICIENCY OF MACHINES

30. Belts and Pulleys.—Belts and pulleys are an important part of shop equipment and it is necessary to understand how to calculate speeds and diameters of pulleys and also lengths of belting.

Pulleys are nothing but gears without teeth and instead of running together directly they are made to drive one another by cords, ropes, cables, or belting of some kind.

As with gears, the speeds of pulleys are inversely proportional to their diameters. That is, if, as in Fig. 90, we have two pulleys A and B which are 24 in. and 12 in. in diam. respectively, the *larger* pulley travels the *slower* and the speed of A is to the speed of B as 12 is to 24.

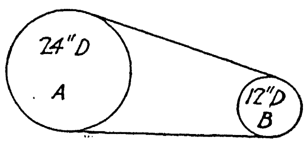


FIG. 90.

This difference between gears running together directly and pulleys driven through belting should be fully understood. A belt "creeps" and "slips" and does not run the driver quite as fast as would be figured out from the diameters of the pulleys, while gears cannot slip and the practical speed and calculated speeds are the same.

When two pulleys are driven through a belt if there is no slipping of the belt, the rim speed of both pulleys is

the same and in Fig. 90 when pulley A makes 1 turn the belt passes over each pulley a distance equal to the circumference of pulley A or $2\pi 12$ or 24π in. and if the rim of pulley B has passed through 24π in. for 1 turn of pulley A and its diameter is 12 in. it has made a number of turns in this time equal to $\frac{24\pi}{12\pi}$ or 2. The following simple rule is convenient for calculating speeds of pulleys.

The diameter of the driving pulley multiplied by its speed is equal to the diameter of the driven pulley multiplied by its speed. In calculating the length of belt to connect two pulleys if the pulleys are the same size and the belting is not crossed the length of belt required is equal to twice the distance between the centers of the pulleys plus the circumference of one of the pulleys. In Fig. 91 the length of belt needed equals

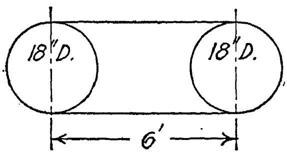


FIG. 91.

$$2 \times 6 + \pi \times \frac{18}{12}$$

or $$12 + 4.7 \text{ or } 16.7 \text{ ft.}$$

If the diameters of the pulleys are not the same, but do not differ greatly and the distance between their centers is large compared with the pulley diameters, the length of belt is found very nearly by adding to twice the distance between the pulley centers half the circumference of each of the pulleys.

PROBLEMS

1. It is desired to run a countershaft at 200 r.p.m. from a line shaft running at 800 r.p.m. If the pulley on the line shaft is 18 in. in diam., what size pulley is required on the countershaft?

2. What length of belt is required to connect two 24-in. pulleys whose centers are 6 ft. 4 in. apart?

3. A belt runs from a 36-in. pulley on a line shaft to a second shaft having a 14-in. pulley. If the line shaft makes 650 r.p.m. what is the speed of the second shaft?

4. Find the length of belt required to connect a 24-in. and a 36-in. pulley whose centers are 8 ft. apart.

5. A main shaft running at 150 r.p.m. with an 18-in. pulley drives a jack shaft through a 12-in. pulley. A second pulley of 24 in. diam. on the jack shaft drives a dynamo through a 12-

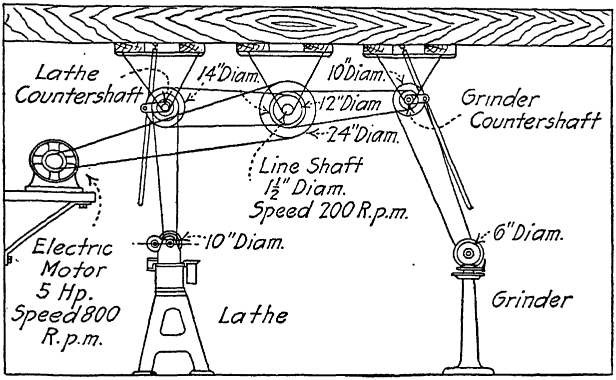


FIG. 92.

in. pulley. Find the speed of the dynamo and make a sketch showing the arrangement of pulleys, and direction of rotation of shafts.

6. Calculate from the following rule the length of belting required to connect two 24-in. pulleys 7 ft. apart if the belt is crossed?

$$l = 2\sqrt{D^2 + S^2} + \pi D$$

When D = diam. of pulleys
S = distance between centers

7. A main shaft running at 200 r.p.m. with a 12-in. pulley drives a second shaft through a 24-in. pulley. The slip of the belt is 2 per cent. of the driving speed. On the second shaft is a second pulley 36 in. in diam. which is connected through a belt to an 18-in. pulley on an electric generator. The belt on the generator

slips 3 per cent. of the driving speed of the second shaft. What is the speed of the generator?

8. Fig. 92 shows the elevation of a layout for a small machine shop, with the equipment as indicated. Working from this sketch, find

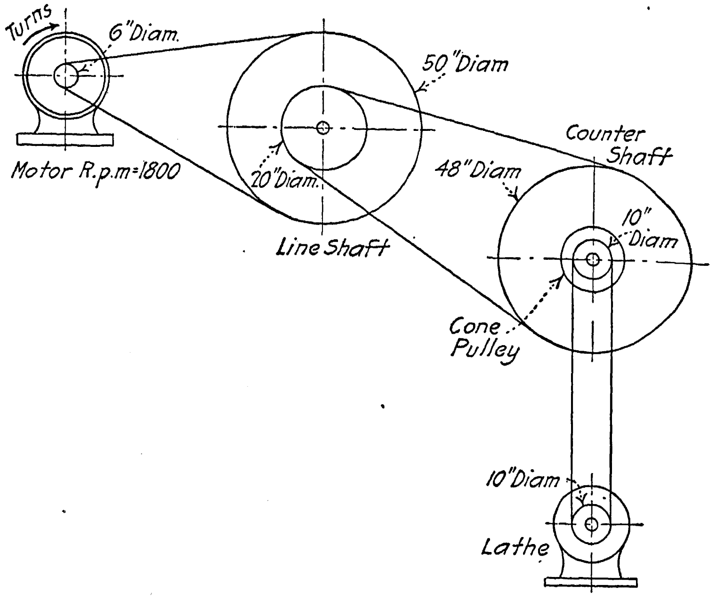


FIG. 93.

1. The diameter of pulley on motor shaft.
2. The speed in r.p.m. of lathe countershaft.
3. The speed of lathe. Pulley on lathe countershaft is 12 in. in diameter.
4. The speed of grinder countershaft. Pulley on grinder countershaft is 8 in. in diameter.
5. The speed of grinder.
6. The weight in pounds of line shaft. Length 10 ft., diam. 2½ in. Material, steel weighing .28 lb. per cu. in.

9. With the belts as arranged in Fig. 93, show the direction of turning of the lathe spindle, with the direction of rotation shown for motor. How could the direction of turning of the lathe be reversed? With data given on sketch what is the speed in r.p.m. of the lathe spindle when belted as shown? Why is it considered better practice to have the tight side of the belt on the bottom of the pulleys?

31. Efficiency of Machines.—In working out the problems on levers, pulleys, inclined plane and so forth we have not taken account of friction or other sources of loss. In other words we have supposed them to be perfect. As actually used their operation is not as satisfactory as our answers show, and as an indicator of the performance of a machine we often find its "efficiency."

$$\text{Efficiency means } \frac{\text{the output of a machine}}{\text{input to the machine}}$$

or better $$\text{efficiency} = \frac{\text{output}}{\text{input}}.$$

The efficiency is a number which shows *how much* of the energy the machine takes, it puts out again as useful work.

Suppose a screw jack has an efficiency of 20 per cent. or .20. This means that it delivers in useful work 20 per cent. of all the energy or work put into it.

If a machine had an efficiency of 100 per cent. or 1 it would mean that efficiency $= 1 = \frac{\text{output}}{\text{input}}$ therefore input $\times$ 1 = output, or output = input. But no machine has an efficiency of 100 per cent. and therefore the output can never equal the input since there are always some losses in the machine itself.

Suppose a 50-h.p. motor has an efficiency of 85 per

cent. when delivering its rated (50) horsepower. To find its input we have

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}, \text{ therefore}$$

$$\text{input} = \frac{\text{output}}{\text{efficiency}} \text{ or input} = \frac{50}{.85} \text{ or } 58.8 \text{ h.p.}$$

If a steam engine delivers 50 h.p. and its input is 70 h.p., its efficiency is found as follows:

$$\text{Eff.} = \frac{\text{output}}{\text{input}} = \frac{50}{70} = .714$$

or $$.714 \times 100 = 71.4 \text{ per cent.}$$

It is therefore to be remembered that in working out a problem on machines we always have to take account of losses that are due to friction in the machines themselves in order to get the correct results.

PROBLEMS

10. An electric generator delivers 48 h.p. and requires 60 h.p. to drive it. What is its efficiency?

11. A screw jack lifts a 20-ton weight 2 ft. thereby doing $20 \times 2000 \times 2$ or 80,000 ft.-lb. of work, and in doing this work requires 334,000 ft.-lb. of work. Find its efficiency.

12. An engine is rated at 100 h.p. and has an efficiency at this load of 75 per cent. What is its input?

13. An electric motor takes 20 h.p. to do a certain amount of work. If its efficiency is 88 per cent., find its horsepower output.

CHAPTER X

MOTION

32. Definitions.—Motion is a change of position and may be of a number of kinds.

In order to set objects in motion forces or pressures are necessary. If a steady force acts on an object moving it, the object increases in motion until the resistance with which the object meets is equal to the force causing the motion and the object then travels over equal distances in equal times or as we say, has a *constant speed.* The word *constant* means unchanging, and *speed*, or what is the same thing, *velocity*, means the rate of motion of an object. If a train going at a steady rate covers fifteen miles in an hour its *speed* or *velocity* is 15 miles per hour (abbreviated m.p.h.). When an object is moving at a constant speed, if the driving force is removed, the speed decreases until the object is brought to rest by resisting forces such as friction, etc. When an object increases in speed it is said to "accelerate," and when it decreases in speed it is said to "retard."

Any object at rest will remain at rest for all time and an object in motion will always move at a *constant speed* in a *straight line* unless it is acted upon by a force or pressure which deflects it or changes its rate of motion, and how much an object will change in speed or direction or both depends on how large the force is acting on it.

When an object is free to fall the force of attraction of

the earth for it or the *force of gravity* causes the object to gain in speed 32.2 *ft. per sec. each second of its fall.*

33. Composition of Motions.—Motions, like forces, may be conveniently represented by straight lines. The length of the line representing to scale the rate of the motion as 10 ft. per sec. or 40 m.p.h. and the direction of the line, the direction of the motion. In Fig. 94 an object at O is represented as having two successive motions of 10 and 15 miles per hour, these motions being 60° apart. The effect of the two motions is the same as *one* motion of 22 m.p.h. in the direction OC. This occurs, for example, if from a train traveling in the direction OB at 10 m.p.h., an object is thrown off in the direction OD at a speed of 15 m.p.h., the object itself is affected by both motions at the same time and travels in the direction OC at the rate of 22 m.p.h. If two trains are traveling in the *same* direction, one at 40 m.p.h. and the other at 35 m.p.h. the *relative* speed of the trains is only 5 m.p.h.

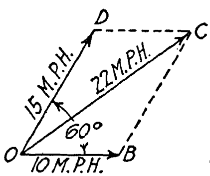


FIG. 94.

If an engine *hauls* a freight train at the rate of 20 m.p.h. and a brakeman runs along the train in the direction of motion at a speed of 5 m.p.h. he is traveling at the rate of 20 + 5 or 25 m.p.h. with respect to the earth or track.

The following rules are convenient in determining speeds.

$$\text{Velocity or speed} = \frac{\text{distance}}{\text{time}}$$

and $$\text{distance} = \text{speed} \times \text{time}$$

$$\text{Acceleration or rate of change of speed} = \frac{\text{speed}}{\text{time}}$$

If a train starts from rest and increases uniformly in speed to 24 m.p.h. in 3 min. it has changed in speed or "accelerated" $\frac{24}{3}$ or 8 m.p.h. each minute. If this constant rate of increase of speed were kept up for 5 min. by a locomotive starting from rest, its speed at the end of the 5 min. would be 8×5 or 40 m.p.h.

It has already been stated that the earth attracting a freely falling object increases its speed about 32.2 ft. per sec. for each second of its fall.

PROBLEMS

1. A locomotive went from Altoona, Pa. to Harrisburg, Pa., a distance of 131 miles in 112 min. What was its average speed in miles per hr., and in ft. per sec.?

2. The same locomotive mentioned in Prob. 1 went from Altoona to Philadelphia, a distance of 236 miles in 3 hr. and 19 min. Making allowance for a 3-min. stop at Harrisburg, find the average speed made in miles per hr. and in ft. per sec.

3. A passenger locomotive leaving a station attains a speed of 40 m.p.h. in 4 min. 30 sec. What is the acceleration or rate of change of speed?

4. An engineer has to make, in 3 hr., a 120-mile run with four 3-min. stops. What average rate of speed does he have to maintain in order to complete the run on time?

5. A passenger train making an average speed of 38 m.p.h. has to make a run of 85 miles. How much sooner will it complete the run than a freight which starts at the same time and makes an average of 25 m.p.h.?

6. The "Pennsylvania Limited" traveling at 60 m.p.h. starts from a station an hour after an accommodation train. The accommodation is delayed ½ hr. and is overtaken by the "Limited in 2 hr. What is the rate of speed of the accommodation train and how far had it traveled when overtaken by the "Limited?"

7. If the 82-in. drivers on a passenger locomotive make 200 revolutions per minute (r.p.m.) what is its speed in miles per hr.? What would be its speed allowing for a 2 per cent. slip?

8. A passenger locomotive with 80-in. drivers making 180 r.p.m. and a freight engine with 60-in. drivers making 150 r.p.m. are traveling in the same direction. What is the relative speed of the locomotives? If the locomotives were traveling in opposite directions what would be their relative speeds?

9. An engineer makes an average of 42 m.p.h. and runs for 4 hr. and 20 min. What distance does he cover?

10. If an object falls freely from rest for 3 sec., what is its speed in ft. per sec. and in miles per hr. at the end of that time?

11. The drivers on a locomotive are 18.85 ft. in circumference and make 250 turns a min. What is the speed of the locomotive in miles per hr.?

12. Two trains start from a station at the same time and travel over level country, the first North at the average rate of 40 m.p.h. and the second East at the average rate of 30 m.p.h. How far apart are the trains at the end of 5 hr.?

13. A train makes a total distance of 236 miles, covering one-third of this distance at the rate of 45 m.p.h. and the remaining distance at the rate of 40 m.p.h. In what time does it make the trip if its total stops take 10 min.?

14. A touring car starts from a garage and reaches a speed of 48 m.p.h. in 4 min. and 20 sec. What is its *acceleration* in miles per hr. per min., and in miles per hr. per sec.?

15. Find the speed in ft. per sec. and in miles per hr. of a freely falling object at the end of the fourth second of its fall.

16. In a trolley car the gear on the driving axle and the gear on the motor axle have a ratio of teeth of 4 : 1. The driving wheel has a diameter of 32 in. Find the speed of the trolley car in miles per hr., when the motor is making 1000 r.p.m.

17. Two trains start from a station at the same time, one going East at 40 m.p.h. for 5 hr. and the other North at 20 m.p.h. for 2 hr. and then East at 10 m.p.h. for 3 hr. Find the distance apart of the trains at the end of the 5 hr.

CHAPTER XI

CUTTING SPEEDS. SPEEDS OF LATHES

34. Definitions and Problems.—The calculation of the circumference and diameter of circles becomes necessary in calculating the cutting speed of work being turned off in a lathe or in calculating the rim speed of fly-wheels, grindstones, etc.

Machines, such as lathes, milling machines, boring mills, planers, etc., are provided with means for changing the speed according to the work turned out. Every mechanic should know how to calculate the proper size of pulleys or gears for any particular job. When a piece of metal of circular section is being turned down in a lathe the cutting tool for each revolution of the work travels a distance (with respect to the metal being turned down) equal to the circumference of the work.

For example, a rod 2 in. in diam. passes a distance of 3.142×2 or 6.284 in. under the cutting tool for each revolution of the rod. If the rod rotated 60 times per min. the cutting speed would be 60 times 6.28 or 376.8 in. per min. or $\frac{376.8}{12}$ or 31.4 ft. per min.

If S is the surface speed in ft. per min.,
N the revolutions per min. of the work,
C the circumference of the work in in.,

the rule is
$$S = \frac{C \times N}{12}$$

For example, a 2-in. diam. brass rod when turned at the rate of 160 r.p.m. has a surface speed of

$$S = \frac{2 \times 3.142 \times 160}{12} \text{ or } 83.8 \text{ ft. per min.}$$

The rim speed of a fly-wheel 72 in. in diam. when running at 200 r.p.m. may be found by the same rule as follows:

$$S = \frac{72 \times 3.142 \times 200}{12} \text{ or } 3770 \text{ ft. per min.}$$

Working the other way round we can find the revolutions per min. (N) at which a wheel must rotate to have a given surface or rim speed.

If a fly-wheel 48 in. in diam. is to have a rim speed of a mile a min. or 5280 ft. per min. its revolutions per min. are found from the rule

$$N = \frac{12 \times S}{C}, \text{ that is}$$

$$\text{Revolutions per min.} = \frac{12 \times \text{speed in ft. per min.}}{\text{circumference of wheel in in.}}$$

Therefore
$$N = \frac{12 \times 5280}{48 \times 3.142} = 420 \text{ r.p.m.}$$

There are average values given for cutting speeds of different metals as follows:

Cutting Speeds in Ft. per Min.

Brass	80
Machinery steel	30
Tool Steel	20
Cast Iron	40

With high-speed steel cutting tools about double the above speeds can be used. No exact figures can be given for the best cutting speeds of the different metals. The

proper speed to use depends on the size of the work, the power and rigidity of the lathe, the kind of tool used and the nature of the work being turned out.

For the surface speed of grindstones the following figures may be taken

For carpenters' tools, 600 ft. per min.
For machinists' tools, 1000 ft. per min.

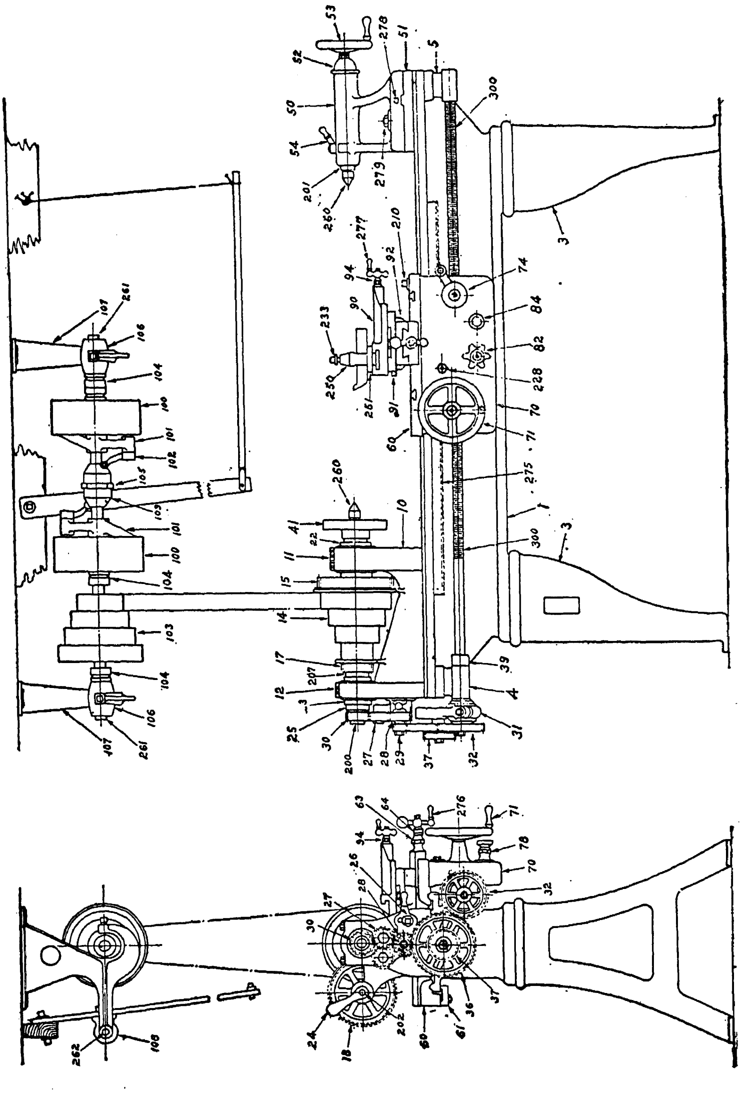
When edge tools are ground care must be taken to avoid too high a speed to prevent heating the edges of the tools and getting rough surfaces. For rapid grinding the following speeds are used:

With Huron stones, 3500 ft. per min.
With Ohio stones, 2500 ft. per min.

Some polishing wheels are run as high as 8000 ft. per min. Emery wheels are run usually at a surface speed of about a mile a min. or 5280 ft. per min.

To find how long it will take the cutting tool on a lathe to travel a given distance along the work with a given feed we proceed as follows:

Suppose a lathe tool has a feed of $\frac{1}{8}$ in. per rev., and we wish to find how long it will take to turn a length of 18 in. on an 8-in. diam. piece with a cutting speed of 60 ft. per min. The circumference of the piece is 8×3.142 or 25.136 in. This is equal to $\frac{25.136}{12}$ or 2.094 ft. With a cutting speed of 60 ft. per min. this requires $\frac{60}{2.094}$ or 28.6 r.p.m. For each revolution with a feed of $\frac{1}{8}$ in. the tool travels along the work $\frac{1}{8}$ of an in. or $28.6 \times \frac{1}{8} = 3.57$ in. per min. To turn a length of 18 in. would therefore require $\frac{18}{3.57}$ or 5.04 min. or a little over 5 min.



Cut of Lathe showing principal parts numbered.

FIG. 95.—Published by permission of the South Bend Lathe Works, South Bend, Indiana, U. S. A.

NUMBER AND NAME OF LATHE PARTS ON DRAWING

NO.

1. Bed.
3. Power Legs.
4. Lead Screw Bracket F.
5. Lead Screw Bracket R.
10. Head Stock.
11. Head Stock Cap, Large.
12. Head Stock Cap, Small.
13. Head Stock Clamp Plate.
14. Spindle Cone.
15. Bull Gear.
16. Bull Gear Clamp.
17. Cone Pinion.
18. Quill Gear.
19. Quill Sleeve.
20. Quill Sleeve Pinion.
21. Ecc. Shaft Bushing.
22. Bronze Box, Large.
23. Bronze Box, Small.
24. Back Gear Lever.
25. Spindle Take up Nut.
26. Reverse Bracket.
27. Reverse Twin Gears (2).
28. Reverse Gear.
29. Stud Gear.
30. Spindle Reverse Gear.
31. Change Gear Bracket.
32. Change Gears.
33. Change Gear Idler.
34. Bushing for Change Gear Idler.
35. Change Gear Collar on L. S.
36. Compound Idler Gear 1 to 2 Large.
37. Compound Idler Gear 1 to 2 Small.
38. Bushing for Compound Idler Gear, 1 to 2.
39. Thrust Collar on Lead Screw.
40. Large Face Plate.
41. Small Face Plate.
42. Turning Gear.
50. Tail Stock Top.
51. Tail Stock Base.
52. Tail Stock Nut.
53. Tail Stock Hand Wheel.

NO.

56. Tail Stock Clamp Plate.
60. Saddle.
61. Saddle Gib.
62. Saddle Lock.
63. Cross Feed Bushing.
64. Cross Feed Gra. Collar.
65. Cross Feed Nut.
66. Plain Rest.
67. Thread Cutting Stop.
70. Apron.
71. Apron Hand Wheel.
72. Lead Screw Half Nut.
73. Lead Screw Half Nut Gib (2).
74. Nut Cam.
75. Nut Cam Washer.
76. Rack Pinion Gear.
77. Auto. Apron Worm Wheel.
78. Auto. Apron Clutch Sleeve Bushing.
79. Auto. Apron Worm Bracket.
80. Auto. Apron Clutch Sleeve.
81. Auto. Apron Clutch.
82. Auto. Apron C. F. Star Knob.
83. Auto. Apron C. F. Lever.
84. Auto. Apron C. F. Lever Knob.
85. Auto. Apron C. F. Gear.
86. Auto. Apron Idler Gear.
87. Auto. Apron Idler Gear Pinion.
90. Compound Rest Top.
91. Compound Rest Swivel.
92. Compound Rest Bottom.
93. Compound Rest End Cap.
94. Compound Rest Bushing.
95. Clmpound Rest Nut.
96. Compound Rest Chip Guard.

Countershaft

100. C. S. Friction Pulleys (2).
101. C. S. Friction Spiders (2).
102. C. S. Friction Fingers (2).
103. C. S. Cone.
104. C. S. Collars (4).
105. C. S. Yoke Lever.

NO.

108. C. S. Shipper Nut.
109. C. S. Yoke Cone.
200. Head Stock Spindle.
201. Tail Stock Spindle.
202. Back Gear Eccentric Shaft.
203. Apron Worm.
204. Apron Rack Pinion.
205. Spindle Sleeve.
207. Spindle Thrust Collar.
208. Apron Worm Collar.
209. Tool Post Block.
210. Carriage Lock Collar Screw.
211. Compound Rest Swivel Bolts.
212. C. G. Bracket Collar Screw.
213. Reverse Collar Screw.
214. Bull Gear Clamp Collar Screw.
215. Apron Clutch Sleeve Hex. Nut.
216. Compound Rest Swivel Stud.
217. Steady Rest Lock Bolt.
218. Auto Cross Feed Lever Stud.
219. Reverse Steel Washer ½″ hole.
220. Apron Clutch Sleeve Pinion.
221. Compound Rest Bottom Gib.
222. Plain Rest Gib.
223. Auto Apron Clutch Screw.
224. Cross Feed Screw.
225. Apron Hand Wheel Pinion.
226. Tail Stock Screw.
227. Reverse Shaft or Stud.
228. Apron Rack Pinion Stud.
229. Reverse Shoulder Screws (2).

NO.

230. Compound Rest Screw.
231. Auto Cross Feed Stud.
232. Apron Half Nut Stud (2).
233. Tool Post Screw.
234. Apron Idler Gear Stud.
235. Cam Cap Screw.
238. Apron Worm Washer.
239. Comp. Rest Steel Wedge.
240. Gap Bridge Pins (2).
241. Reverse Stud Collar ¾″ hole.
242. Change Gear Spindle Knob.
250. Tool Post.
251. Tool Post Ring.
252. Tool Post Wedge.
253. Tool Post Wrench.
254. Compound Rest Wrench.
257. Compound Rest Top Gib.
258. Comp. Rest C. P. Headless Set Screws.
260. Centers (2).
261. C. S. Shaft.
262. C. S. Shipper Rod.
263. C S. Expansion Wedges.
275. Rack.
276. Cross Feed Ball Crank.
277. Compound Rest Handle.
278. Tail Stock Set Over Screws (2).
279. Tail Stock Clamping Bolt, Nut and Washer.
300. Lead Screw.

PROBLEMS

1. A steel bar 3 in. in diam. is to be turned in a lathe. If the cutting speed is to be 30 ft. per min. at how many revolutions per min. should the bar turn?

2. A cast-iron pulley 18 in. in diam. is to have its rim face turned down in a lathe. With a cutting speed of 40 ft. per min. what should be the revolutions per min. of the pulley?

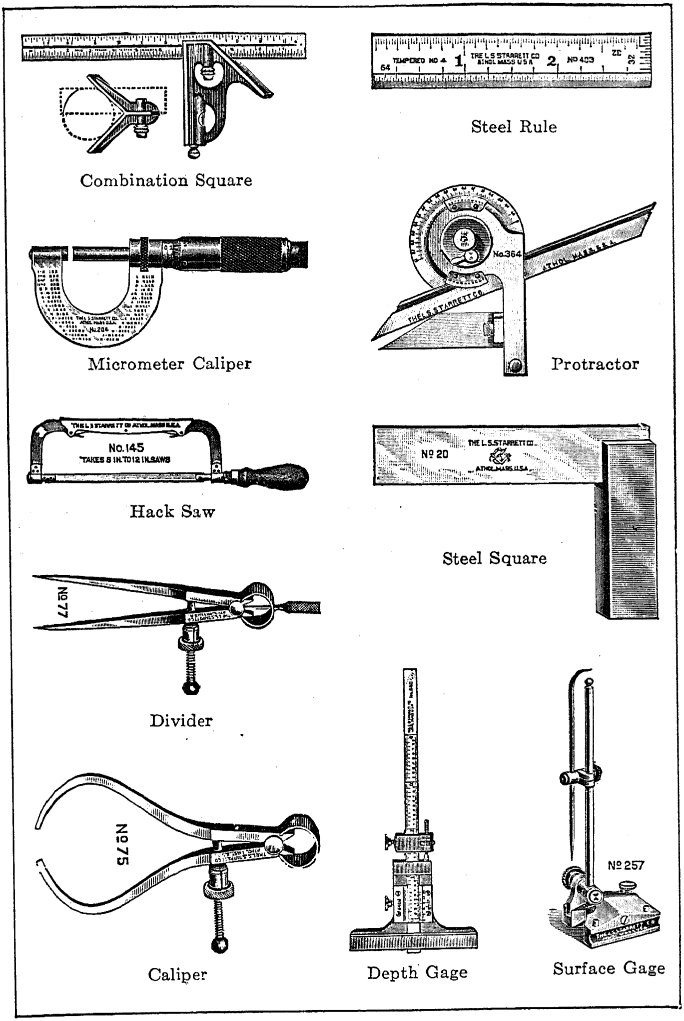


FIG. 96.—Cut showing machinist tools.

3. An emery wheel 14 in. in diam. runs at 1380 revolutions per minute (r.p.m.). Find its surface speed in ft. per min.

4. What is the speed of a belt running on a 3-ft. pulley which makes 300 r.p.m.?

5. At how many revolutions per min. should we run a 2-in. diam. brass rod in a lathe for a cutting speed of 80 ft. per min.?

6. Find the revolutions per min. for a $1\frac{1}{8}$-in high-speed drill to give a cutting speed at the outer edge of 80 ft. per min. If the feed is .015 in. per rev., how long will it take to drill through $1\frac{1}{2}$ in. of metal?

7. Find the revolutions per min. of an 18-in. polishing wheel whose surface speed is 8000 ft. per min.

8. At how many revolutions per min. should a 30-in. grindstone be run to obtain the proper surface speed for grinding machinists' tools?

9. A piece of work 6 in. in diam. is being turned down at a cutting speed of 60 ft. per min. (f.p.m.). If the cutting tool feeds along the work $\frac{1}{8}$ in. for each revolution, how long will it take to turn a length of 8 in.?

10. A steel crank shaft is to be finished to a diameter of 8 in. With a cutting speed of 40 f.p.m. what should be its speed in revolutions per min.?

CHAPTER XII

VOLUME AND PRESSURE OF GASES

35. Definitions and Problems.—A very important law has been discovered in regard to the relation of the volume, temperature and pressure of gases. Steam follows this law very closely. It has been found that if the temperature of a gas does not change, its volume is practically "inversely proportional" to its pressure, or its volume times its pressure is a constant quantity. This means that if we have a cylinder, as in Fig. 97, which contains 10 cu. ft. of air with the piston in position *A* and the gauge reads 10 lb., making an absolute pressure of 10 + atmospheric pressure (that is, 10 + 14.7 or 24.7 lb.), and we move the piston to the position *B* so that the volume is only 5 cu. ft. or *one-half* of its former value, the absolute pressure becomes 2 × 24.7 lb. or 49.4 lb. or *twice* its former value and the gauge reads 49.4 − 14.7 or 34.7 lb. since the gauge is made to read pressures above that of the atmosphere. That is, making the gas occupy half the volume has doubled its pressure. If, on the other hand, we *increase* the volume of the gas, we *lower* its pressure. This is exactly what happens in a steam engine cylinder into which steam enters at perhaps 200-lb. pressure, then expands, does work

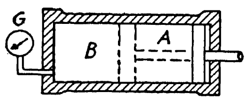


FIG. 97.

in moving the piston, becomes reduced to a comparatively low pressure and exhausts into the air.

The principle of expansion and compression of gases is made extensive use of in all steam and gas engines and in air-braking devices. The rule is: If the temperature of a gas does not change, the product of its volume and pressure remain practically the same throughout a considerable range of pressure.

PROBLEMS

1. If the air in a cylinder containing 5 cu. ft. at atmospheric pressure is compressed to a volume of 1 cu. ft., what is its pressure?

2. A gas occupies 10 cu. ft. at an atmospheric pressure of 15 lb. per sq. in. and is afterward compressed to a volume of 2 cu. ft. What is its pressure?

3. During the movement of a piston in a steam cylinder the pressure drops from 90 lb. to 50 lb. If this change took place according to the law explained, what change of volume took place?

4. A 30-in. × 36-in. cylinder with a .2 cu. ft. clearance volume at each end takes in air at atmospheric pressure of 14.7 lb. What is the pressure of this air when the piston has completed one-half the return stroke and is compressing the free air drawn in on the forward stroke?

5. Twenty cubic feet of free air is compressed to a pressure of 60 lb. gauge. What volume does it then occupy?

6. The air cylinder of an air compressor is 20 in. × 28 in. and its clearance volume is .2 of a cu. ft. What is the pressure of the discharged air if the outlet valves lift at ¾ stroke?

7. If the admission of steam to a 24-in. × 26-in. cylinder at a pressure of 200 lb. per sq. in., gauge is cut off at 50 per cent. of the piston stroke (½ stroke) what is the terminal pressure, that is, the pressure at the end of the stroke?. (Give the answer in gauge pressure.) Clearance volume at each end of cylinder is .2 cu. ft.

8. What would be the terminal pressure if the cut-off occurs at 25 per cent. of the stroke (¼ stroke) with the conditions of Prob. 7?

9. With a back pressure on the piston during the exhaust stroke of 3 lb. per sq. in. gauge, what would be the compression pressure (lb. per sq. in. gauge) at the end of the stroke when compression or exhaust closure occurs at .6 of the piston stroke, in a 24-in. × 26-in. cylinder? Clearance volume at end of cylinder is .2 cu. ft.

10. What would be the compression pressure (lb. per sq. in. gauge) if compression of the steam occurs at $\frac{7}{8}$ stroke with other conditions the same as in Prob. 9?

CHAPTER XIII

WORK AND POWER.

36. Definitions and Rules on Work.—Work means the overcoming of resistance of any kind. If you drag an object along the floor you do work in overcoming the friction between the object and the floor. In lifting an object you do work against gravity which tends to pull the object toward the earth. Steam in a locomotive cylinder does work when it expands and moves the piston against the resisting forces. "Work" is measured in "foot-pounds," that is, the *product of the resistance overcome in lb.* and the *distance in ft. through which it is overcome.* If you lift a 10-lb. weight 5 ft. you do 10×5 or 50 *ft.-lb.* of work against gravity.

If a train weighs 350 tons and the resistance offered to its motion is 12 lb. per ton, the work done in moving the train 1 mile at uniform speed is

$$350 \times 12 \times 5280 = 22{,}176{,}000 \text{ ft.-lb.}$$

If it takes an average pressure of 30 tons to punch a $\frac{3}{8}$-in. steel plate, the work done equals the resistance in lb., or 30×2000 lb. times the thickness of the plate in ft., $\frac{3}{8}$ in. $= \frac{1}{32}$ ft.

$$\begin{aligned}\text{Therefore work done} &= 30 \times 2000 \times \tfrac{1}{32} \\ &= 1875 \textit{ ft.-lb.}\end{aligned}$$

37. Power, Horsepower.—*Power* means the *rate* at which work is done and is expressed in *foot-pounds per*

min. or in foot-pounds per sec. When 550 ft.-lb. of work are done in a sec. or 33,000 ft.-lb. in a min. 1 h.p. is represented. A 10-horsepower (abbreviated h.p.), engine therefore is one which can do 33,000 × 10 or 330,000 *ft.-lb. of work in* 1 *min.*, or is one capable of overcoming a resistance of 330,000 lb. through a distance of 1 ft. *in* 1 *min.*

The rule for horsepower is:

$$\text{Horsepower} = \frac{\text{pounds} \times \text{feet}}{550 \times \text{seconds}}$$

or

$$\text{Horsepower} = \frac{\text{pounds} \times \text{feet}}{33{,}000 \times \text{minutes}}$$

In electrical terms 1 h.p. is equal to 746 watts. The horsepower required in any case depends as much on the *time* in which the work is to be done as on the *amount* of work to be done.

PROBLEMS

1. What is the horsepower of an engine that will raise 8000 lb. of water 200 ft. in 5 min.?

2. What is the horsepower of an automobile engine which can do 82,500,000 ft.-lb. of work in 5 min.?

3. What horsepower motors are required to raise a 222,000-lb. locomotive 4 ft. in 1 min.?

4. If a man in a day of 8 hr. raises 20,000 lb. of gravel through an average distance of 5 ft., what horsepower represents his working power?

5. A man weighing 150 lb. runs up stairs to a vertical height of 20 ft. in 2 min. What horsepower is he exerting to raise his own weight?

6. What is the useful horsepower of an engine which draws a train of 200 tons on a level at the uniform rate of 30 miles per hr. against a resistance of 16 lb. per ton weight?

7. A 7-h.p. engine is used to raise a weight of 24 tons. How high will it raise it in 1 min.? If it is desired to raise the weight in twice the time, what horsepower is required?

8. It is desired to raise a weight of 5 tons 200 ft. in 40 min. What horsepower is necessary to do the work?

9. What average horsepower is required merely to lift a 250-ton train up 5 miles of grade rising 1 ft. in every 528 ft., if the train is moving at the rate of 25 miles per hr.?

10. If the name plate on a motor gives the motor rating as 5 h.p., how many foot-pounds of work should the motor do per min. without overheating? Can the motor be worked above its rated horsepower?

11. From the rule that horsepower per cylinder $= \frac{PLAN}{550}$ in which P is the average steam pressure in pounds per sq. in., L the length of stroke in ft., A the area of the piston in sq. in., and N the number of strokes per sec., find the horsepower developed on both sides of a locomotive whose cylinders are 20 in. $\times$ 28 in., working with an average steam pressure of 180 lb. per stroke and 150 strokes per min.?

12. A 12-ton casting is raised 20 ft. in 1 min. by a traveling crane. What horsepower does the motor on the crane exert?

13. From the rule for horsepower per cylinder, h.p. $= \frac{PLAN}{33{,}000}$ in which P is the average pressure in pounds per sq. in. on the piston, L the length of stroke in feet, A the area of piston in square inches, and N the strokes per min. (or 2 times the r.p.m.) find the horsepower for both cylinders of the following locomotive. Cylinders 24 in. $\times$ 26 in. (24 in. diam. of piston and 26 in. stroke of piston), average steam pressure 160 lb. per sq. in., and running at 200 r.p.m.

14. From the rule in the foregoing problem calculate the total cylinder horsepower of a 4-cylinder Mallet locomotive.

Cylinders 27 in. $\times$ 28 in., average steam pressure on piston 130 lb. per sq. in., and r.p.m. 150.

15. Find the horsepower of a freight locomotive running at 10 m.p.h. with an average cylinder pressure of $\frac{4}{5}$ boiler pressure. Drivers 62 in. in diam., cylinders 24 in. $\times$ 28 in., boiler pressure 205 lb. per sq. in.

The power delivered by an engine or a motor at its pulley, that is, the actual power available for doing work may be found for small machines by the Prony-Brake Method.

Fig. 98 shows such a brake consisting of shoes s and s_1, which are clamped to the machine pulley by bolts b and b, which may be regulated to increase or decrease the pressure of the shoes on the pulley. When the pulley rotates in the direction of the arrow, the entire brake and lever L tend to rotate in the same direction. This tendency is counterbalanced by the pull on the spring shown, from which we may read the pounds pull for different degrees of pressure of brake shoes on pulley.

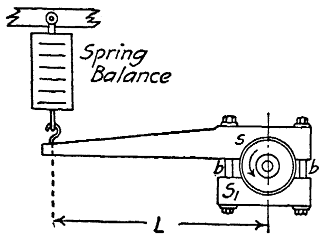


FIG. 98.

The spring keeps the lever arm practically horizontal throughout the test. The power absorbed by the brake shoes "s" and "s_1" equals the amount of work done by the revolving pulley on the machine. In using the brake we first take the reading of the spring balance with the machine not running in order to account for the weight of the lever itself, then after bringing the machine up to speed and putting a "load" on it by means of bolts b and b, the balance reading is again taken, and the initial reading subtracted to get the net pull due to the rotation alone. If P is the pull in pounds on the balance, L the length of the lever as shown and r.p.m. the revolutions per min. of the pulley, the *foot-pounds* of work done *per min.* equals $P \times 2\pi L \times$ r.p.m., and the horsepower equals

$$\frac{P \times 2\pi L \times \text{r.p.m.}}{33{,}000}.$$

When used in tests of this kind the machine pulley is made so that water may be run on the inside of the

rim to keep the pulley cool and absorb the heat produced by the transformation of the mechanical work done at the pulley.

The principle of the brake described above is the same as that of the power of a belt in which the effective belt pull is "P" pounds on a pulley of L radius and running at a speed represented by the symbol r.p.m. In this case the belt moves at a speed of $2\pi L \times$ r.p.m. feet per min. and the horsepower is therefore $\dfrac{P \times 2\pi L \times \text{r.p.m.}}{33{,}000}$ or the same rule as derived above.

The principle of the brake may also be considered the same as that of a weight "P" being raised by a rope wound on a pulley of radius L, as in Fig. 99.

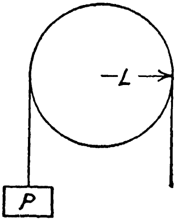


FIG. 99.

Example.—Find the horsepower delivered by a direct-current motor in a brake test in which the net effective pull at end of lever 2½ ft. long is 10 lb., with the pulley rotating at 900 r.p.m.

Solution.—Horsepower $= \dfrac{P \times 2\pi L \times \text{r.p.m.}}{33{,}000}$

or

$$\frac{10 \times 2 \times 3.14 \times 2.5 \times 900}{33{,}000}$$

which equals 4.282 or practically 4.28 h.p.

PROBLEMS

16. Find the horsepower of an engine whose shaft rotating at 90 r.p.m. produces a pull of 1800 lb. at the end of a brake arm 9 ft. long.

17. In a brake test on a motor it was found that with a speed

of 750 r.p.m., a pull of 20 lb. was produced at the end of a 5-ft. lever. What horsepower did the motor deliver?

To find the pull produced at the end of a given lever at a given speed and horsepower we have by transposing the horsepower rule,

$$P = \frac{\text{horsepower} \times 33{,}000}{2\pi L \times \text{r.p.m.}}$$

Writing this equation for the length of lever arm "L" we have

$$L = \frac{\text{horsepower} \times 33{,}000}{P \times 2\pi \times \text{r.p.m.}}$$

and for the revolutions per min., we have

$$\text{Revolutions per min.} = \frac{\text{horsepower} \times 33{,}000}{P \times 2\pi L}$$

These rules enable us to find any one of the variable quantities in the rule in terms of the other quantities.

PROBLEMS

18. What pull "P" is produced on the end of a 4-ft. brake lever when the machine pulley is making 250 r.p.m. and machine is delivering 8 h.p.?

19. Find the brake horsepower of a steam engine running at 250 r.p.m. and exerting a pull of 80 lb. at the end of a lever 6 ft. long?

20. What length of lever arm is required for a pull of 18 lb., with a speed of 180 r.p.m. and a load of 2 h.p.?

The *brake horsepower* is the power of an engine or motor or water wheel available for use at the machine pulley and does not include the power lost in the machine itself through friction.

If in a steam engine we found the horsepower developed in the *cylinder* and called the *cylinder* or *indicated* horsepower we would find that this power is greater than the *brake* horsepower by the amount of power lost in friction in the engine mechanism between the cylinder and the fly-wheel or pulley. The horsepower thus lost is called the *frictional horsepower*.

In finding the *indicated* or cylinder horsepower of an engine we use a steam indicator or as it may be called, a recording steam gauge which takes a record of the steam pressure in the cylinder throughout the stroke of the piston. Such a card diagram or record is shown in Fig. 100. In making a test indicator cards are taken at each end of the cylinder. From these we find the average pressure throughout the stroke by finding the area of the card, dividing this by the length of the card, and multiplying by the constant of the indicator. For

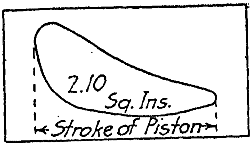


FIG. 100.

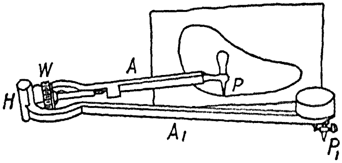


FIG. 101.

example, if the card shown has an area of 2.10 sq. in. and its length is 3.73 in., the average height $= \frac{2.10}{3.73}$ or .563 in. If the scale of the diagram is 80 lb. to the inch this represents an average pressure of $.563 \times 80$ or 45 lb. per sq. in.

From this average pressure called the *mean effective pressure* we calculate the horsepower from the rule

$$\text{Horsepower} = \frac{PLAN}{33,000}.$$

In which P is the mean effective pressure.
L is the length of stroke in ft.
A is the area of the piston in sq. in.
N is the number of strokes per min.

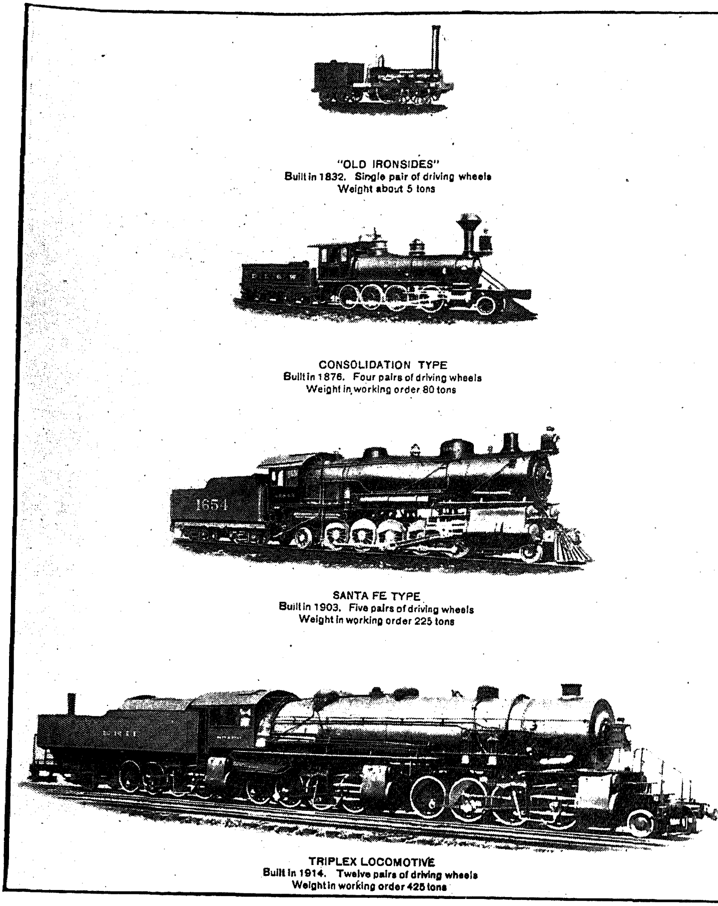


FIG. 102.—Cut showing the development in size and weight of locomotives.

To find the area of the indicator card shown, in Fig. 100 we make use of a *planimeter* or *plane meter* as shown in Fig. 101. This consists of the two arms A and A_1 hinged at H and with pin points at P and P_1. The point P_1 is fixed and the point P is moved around the outline of the card as shown, while the wheel W moves and carries a scale which indicates directly the number of square inches in the area traced around when the point P has completed the outline.

CHAPTER XIV

CALCULATION OF BELTING. ENERGY

38. Horsepower of Belting.—When two shafts are too far apart to connect them by gears for transmitting power, belts are used. The power which a belt transmits depends on the effective pull tending to turn the driven pulley and the speed with which the belt is traveling. The pull which a belt can safely stand depends on its material, width and thickness. They are sometimes made of one, two, three or four thicknesses of leather and are called single, double, triple or quadruple. The widths of belts vary from about an inch to 2 ft. depending on the amount of power transmitted. Whether single, double or other thickness of belting is used depends largely on the size of pulleys on which the belt runs. Practice shows that single belts are best on pulleys less than a foot in diameter, and double belts on pulleys a foot or larger in diameter. On main drives or in cases where the amount of power transmitted would cause an excessively wide double belt a triple or quadruple belt is used.

It has been found that single leather belts will operate satisfactorily with an allowable pull of about 30 lb. per in. of width. For a double belt about 65 or 70 lb. is allowed per in. of width. Almost any mechanical handbook contains figures taken from the results of practice for the best allowable sizes of belts.

Practice shows that the linear speed of belts should not exceed 4500 ft. per min. if the belt is to make good contact with the pulleys. As usually operated, speeds below this value are used.

If P is the allowable pull per in. of width of belt, W the width of belt in in., the pull times the width, that is $P \times W$, gives the total effective pull or the force transmitted by the belt. This force *in lb.* multiplied by the speed of the belt in *ft. per min.* gives the *foot-pounds* of work per min. transmitted by the belt.

To obtain horsepower we divide foot-pounds per min. by 33,000, since 1 h.p. is the performance of 33,000 ft.-lb. of work per min. If S is the speed of the belt in ft. per min. and using the letter given above for the effective pull on the belt, we have the rule

$$\text{Horsepower transmitted by belt} = \frac{P \times W \times S}{33{,}000}.$$

Example.—Find the horsepower transmitted by a belt 5 in. in width, if the allowable pull per in. of width is 35 lb. and the speed of the belt 2400 ft. per min.

Solution:

$$\text{Horsepower} = \frac{P \times W \times S}{33{,}000} \quad \text{or} \quad \frac{35 \times 5 \times 2400}{33{,}000}$$

which equals 12.7.

To find the width of belt to transmit a given horsepower for a given belt speed, we transform the above equation to express W in terms of the other quantities involved as follows.

If H represents the horsepower transmitted we have as above, $H = \frac{P \times W \times S}{33{,}000}$ and therefore $W = \frac{H \times 33{,}000}{P \times S}$.

Example.—Find the width of double leather belt re-

quired to transmit 100 h.p., if the speed of belt is 4000 ft. per min. and the allowable pull per in. of width is taken as 70 lb. since this is a double belt.

Solution:

$$W = \frac{H \times 33{,}000}{P \times S} \text{ or } \frac{100 \times 33{,}000}{70 \times 4000} \text{ which equals } 11.8$$

in. Therefore a 12-in. belt should be used.

PROBLEMS

1. Find the width of a single belt required to transmit 95 h.p. when belt runs on a 14 ft. diam. pulley at 70 r.p.m. when $P = 40$ lb.

NOTE.—The speed of the belt in ft. per min. equals the product of the circumference of the pulley in ft. by the r.p.m. of the pulley.

2. Find the width of belt required to transmit 15 h.p., if belt speed is 3500 f.p.m. and allowable pull per in. of width of belt is 50 lb.

39. Definitions on Energy, and Problems.—Energy is power to do work.

All objects possess *energy* on account of work having been done upon them at some time. Energy, like work, is measured in *foot-pounds.* There are, in general, two kinds of energy due either to the *motion* or to the *position* of objects. *Energy of motion* is called *kinetic energy* and *energy of position* is *potential energy.* For example, an object set in motion can overcome a certain amount of resistance before being brought to rest, and the energy which the object has on account of its motion is used up in overcoming the resistance, bringing the object to rest. Fly-wheels on engines both receive and give up energy and thus cause the engine to run more smoothly throughout the stroke.

Elevated weights have power to do work on account

of their elevated position, as in various types of hammers, pile drivers, etc. The rule for finding the kinetic energy (*K.E.*) of an object is:

$$K.E. = \frac{Wv^2}{64.4}.$$

In which W is the weight of the object in lb., v is its velocity or speed in ft. per sec. Thus a weight of 230 lb. moving with a speed of 1200 ft. per sec. has a kinetic energy of $\frac{230 \times 1200 \times 1200}{64.4}$ or approximately 5,140,000 ft.-lb. The rule for potential energy (*P.E.*) is:

$$P.E. = W \times H$$

W is the weight of the object in lb. and H is its height in ft. above a given point. The energy is then given in foot-pounds. For example: A 2-ton steam hammer when raised 4 ft. has a potential energy of $2 \times 2000 \times 4$ or 16,000 ft.-lb. on account of its position and if let fall it would do 16,000 ft.-lbs. of work.

PROBLEMS

3. A weight of ½ ton is moving with a speed or velocity of 5 ft. per sec. How much energy in foot-pounds is stored up in it?

4. The rim of a fly-wheel weighing 3 tons moves with a speed of 20 ft. per sec. How many foot-pounds of energy are stored up in it?

5. A 3-ton hammer is raised and dropped a distance of 8 ft., 8 times per min. How much work does it do per min.?

6. Three thousand gallons of water are pumped into a tank an average height of 10 ft. above the level of a track. What energy does this body of water have with respect to the track?

7. If a punch makes a hole through a ½-in. steel plate and meets with an average resistance of 30 tons, how much energy is consumed in the punching alone?

8. A machinist wields a 14-lb. hammer and strikes with a veloc-

ity of 30 ft. per sec. and 28 blows per min. How many foot-pounds of energy are produced per min.? At what horsepower does he work?

9. How much work in foot-pounds is required to raise a 10-ton casting vertically through a height of 5 yd.?

10. If 10,000 cu. ft. of water flow per min. over a fall 20 ft. high, how many foot-pounds of energy does this water deliver at the foot of the fall? (1 cu. ft. of water weighs 62.5 lb.)

CHAPTER XV

HEAT

40. Definitions.—Heat is one form of energy. Other forms of energy such as electrical, chemical, energy of position and of motion can be changed into heat. Heat is produced by friction, by chemical action, and by the electric current flowing through a *resistance* such as copper or iron wire. In lighting a match heat is produced both by *friction*, and by *chemical action* in the burning of the match. The heat produced by a cutting tool on the metal being turned off in a lathe, and also that produced in grinding tools, are familiar examples of heat produced by friction, or in other words, by the change of *mechanical work* into *heat*. The burning of fuel is an example of a *chemical* change involving heat action. Objects in motion become heated when brought to rest by friction or by striking other objects.

41. Temperature.—The temperature of an object means not only whether the object is "hot" or "cold," but whether it takes heat from, or gives heat to, surrounding objects. When two objects at different temperatures are brought together there is a tendency toward an equalization of temperature. Like water which flows from a point of high to one of low level, heat flows from an object of high to one of low temperature. For measuring temperature, in most cases for ordinary work,

mercury thermometers are used, either the Fahrenheit or the Centigrade as shown in Fig. 103.

The Centigrade thermometer has 100 equal divisions on its scale between the point of melting ice and boiling water, with the 0 mark at the point of melting ice.

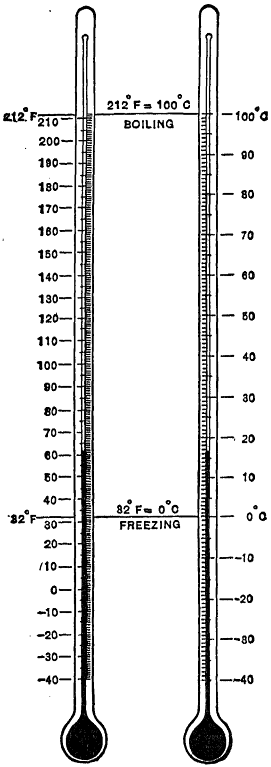


Fig. 103.

The Fahrenheit thermometer has 180 equal divisions on its scale between the points where ice melts and water boils. The 32-degree mark is placed at the point where ice melts and the 212-degree mark at the point where water boils.

To change from a temperature in degrees Fahrenheit to the corresponding temperature in degrees Centigrade, and vice versa, the following rules are used.

$$F.^\circ = C.^\circ \times \frac{9}{5} + 32.$$

$$C.^\circ = (F.^\circ - 32)\frac{5}{9}$$

For example, using the first rule, suppose we wish to find the temperature Fahrenheit corresponding to 30° Cent., we have

$$F.^\circ = 30^\circ \times \frac{9}{5} + 32$$

or $\quad F.^\circ = 54 + 32$ or 86°.

That is, 86° on the Fahrenheit thermometer correspond to 30° on the Centigrade thermometer.

Working the other way round and using the second rule

$$30 = (\text{F.} - 32)\frac{5}{9} \text{ or } 30 = \frac{5\text{ F.}}{9} - \frac{160}{9}$$

or $\quad 270 = 5\text{ F.} - 160 \text{ and F.} = 86°.$

Alcohol and air thermometers are also used for measuring temperatures and these as well as the mercury thermometers make use of the principle of expansion and contraction of the alcohol, air or mercury with different temperatures, to obtain different readings.

42. Heat Measurement.—Heat is measured in *British Thermal Units* (abbreviated B.t.u.). One B.t.u. is the amount of heat required to raise the temperature of 1 lb. of water 1 ° Fahr.

For example, if 10 lb. of water are heated from 60° Fahr. to 90° Fahr. the amount of heat taken up by the water equals 10 × (90° − 60°) or 10 × 30 or 300 B.t.u.

When water reduces in temperature it of course gives up heat and the calculations are made in the same way by multiplying the weight of water in lb. by the drop in temperature in degrees Fahrenheit. This gives the number of B.t.u. liberated by the water. It should be understood that temperature means heat "intensity" or as we might say heat "pressure" and is measured in "degrees," while "quantity" of heat is measured in B.t.u. We can make the same comparison with air, in which the "intensity" or "pressure" is measured in "lb." by a gauge, while the "quantity" of air is measured in cu. ft. Again in the case of steam, we measure the "pressure" in "lb." and the "quantity" in lb. of condensed steam.

If two iron castings are of different weights, it will take

a different "quantity" of heat in B.t.u. to heat them to the same "intensity" or "temperature."

In the same way two air cylinders can contain different quantities of air and still both be at the same pressure.

If the change of temperature is given in Centigrade rather than in Fahrenheit degrees we multiply the number of Centigrade degrees by $\frac{9}{5}$ since 1 Centigrade degree equals $\frac{9}{5}$ Fahrenheit degrees.

If the weight is given in units other than pounds we first change the weight to the equivalent number of pounds.

Example.—How much heat in B.t.u. is taken up by 1 ton of water which is heated from 10° Cent. to 40° Cent.?

Solution.—B.t.u. $= 1 \times 2000 \times (40-10) \times \frac{9}{5}$ or $2000 \times \frac{30 \times 9}{5}$ which equals 108,000. *Ans.*

43. Expansion and Contraction Due to Heat.—Most substances expand or increase in volume when heated and contract or decrease in volume when heat is removed from them.

This effect takes place in solids, liquids, and gases. The principle of expansion and contraction is made of considerable use and in some cases it may cause considerable trouble unless allowed for.

When a piece of metal, for example, is heated, it expands in all directions. The expansion in one direction or dimension is that with which we are particularly concerned and is called the "linear" expansion of the substance.

For example, in the case of mercury thermometers the principle of expansion is used, also in steam piping,

allowance must be made for the lengthening and shortening of the pipe system due to heat changes. In laying steel rails an allowance is made between the ends of the rails for contraction and expansion. The same is true in the case of bridges and other metal structures.

The power of expansion and contraction of metal is very great and if allowance is not made for it the expansion will take place just the same with the result that the part expanding will buckle or give itself the necessary room for expansion in one way or another. These characteristics of metals are made use of in putting tires on wheels, in making shrink fits and numerous other applications.

In the case of tires put on wheels and shrink fits of various kinds, an allowance is made per in. of diameter of the object on which the fit is to be made of approximately $\frac{1}{1000}$ of an in. and in the case of a locomotive driver 68 in. in diam. the tire would be turned 68 times $\frac{1}{1000}$ or $\frac{68}{1000}$ of an in. smaller in diam. than the center on which it is to go. It is then heated so that it will expand sufficiently to slip on to the wheel before it cools. As it cools it contracts and grips the wheel. As it cannot return entirely to its original size this gripping power is sufficient to hold the tire in place during severe service. In case of a crank to be fitted on a 6-in. shaft the crank should be turned about $\frac{6}{1000}$ of an in. smaller than the shaft or else the shaft made $\frac{6}{1000}$ of an in. too large and the crank made just 6 in. in diam. This, as explained will make the proper allowance so that the crank will grip the shaft tightly after it has been shrunk on.

The amount of expansion of an object depends upon the kind of material, upon its length and upon the change in temperature of the object. Tables are prepared, which

give us for different materials the amount of expansion per unit length, per degree change in temperature. These figures are called "coefficients of linear expansion."

The following figures for some of the most used materials are sufficiently accurate for our purposes.

COEFFICIENTS OF EXPANSION OF SOLIDS

Metal	Coefficient for 1° Fahr.
Aluminum (cast)	.00001234
Brass, cast	.00000957
Brass, plate	.00001052
Concrete: cement, mortar and pebbles	.00000795
Copper	.00000887
Iron, wrought	.00000648
Iron, cast	.00000556
Steel, cast	.00000636
Steel, tempered	.00000689

The coefficient for 1 Centigrade degree of temperature may be obtained from those above by multiplying them by $\frac{9}{5}$.

To calculate the amount of expansion the rule is, multiply the coefficient of expansion by the length of the object and by the number of degrees rise in temperature. If the length is taken in ft. the expansion will be in ft. If the length is in in. the answer will be in in., etc.

Example.—What is the amount of expansion of an 18-ft. brass rod when heated 150° Fahr.?

Solution.—Amount of expansion = Coefficient of expansion $\times$ length of rod $\times$ degrees rise in temperature.

$$\text{Expansion} = .0000096 \times 18 \times 150 \text{ or } .0259 \text{ ft.}$$

Example.—What will be the increase in diam. of a 64-in. diam. locomotive driver when heated 500° Fahr.?

Solution.—The coefficient of expansion of steel is taken

as .0000064. The expansion therefore equals .0000064 × 64 × 500 or .2048 in.

Heat also causes liquids and gases to expand, but in the case of gases the change of volume depends upon the pressure of the gas as well as upon its temperature. Solids are liquefied by heat as in melting iron and steel and liquids are changed into vapors as in the production of steam in the locomotive boiler.

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701

PROBLEMS

1. Find the expansion in an aluminum rod 20 ft. long if it is heated 750° Fahr.

2. What is the approximate expansion in the length of a steel bridge 2800 ft. long through a difference of temperature of 0° Fahr. in winter and 90° Fahr. in summer?

3. The walls of a building 40 ft. wide were drawn in and straightened by using iron rods through them, placing plates on the ends of the rods, heating the rods, tightening up the plates by means of nuts and allowing the rods to cool and contract, thus drawing the building together. If the rods each time changed in temperature 800° Fahr. how much was the building drawn together with each heating and tightening of the plates?

4. What is the amount of expansion in a 200-ft. span of copper wire which undergoes a change of temperature of 80° Fahr.?

5. An iron steam pipe changes in temperature from 180° Fahr. to 8° Fahr. What allowance must be made for the change in length if the pipe is 160 ft. long at 180° Fahr.?

6. If steel girders on a bridge are 60 ft. long at 20° Fahr., what is their length at 94° Fahr.

7. What allowance must be made for a crank to be fitted to a 5-in. diam. shaft?

8. Find the expansion in a 40-ft. rail between the temperature of − 20° Fahr. and 90° Fahr.

44. Specific Heat.—The specific heat of a substance means the number of heat units (B.t.u.) required to raise the temperature of any weight of the substance

1°, as compared with the number of heat units (B.t.u.) required to raise the temperature of the same weight of water 1°.

The following are examples of specific heat: Steam .48, iron .114, lead .0314.

This means that to raise the temperature of 1 lb. of iron 1° Fahr. requires only .114 B.t.u., whereas to raise the temperature of 1 lb. of water 1° Fahr. requires 1.0 (one) B.t.u.

45. Latent Heat.—Latent or "hidden heat" is that required to change the *condition* of a substance from a solid to a liquid or from a liquid to a vapor without changing its temperature.

To vaporize 1 lb. of water at 212° Fahr. to steam at 212° Fahr. at ordinary pressure requires 966 B.t.u. To melt 1 lb. of ice at 32° Fahr. to water at 32° Fahr. requires 142 B.t.u. of heat.

46. Mechanical Equivalent of Heat.—The mechanical equivalent of heat means the relation between units of work in foot-pounds, and equivalent *heat units*. We all know that friction produces heat and to overcome the friction work must be done. When 778 ft.-lb. of work are done, an amount of heat is produced equal to 1B.t.u. and this amount of heat is sufficient to raise the temperature of 1 lb. of water 1° Fahr.

Example.—How much heat is produced by 10,000 ft.-lb. of work?

Solution: $\frac{10{,}000}{778}$ or 12.8 B.t.u. *Ans.*

47. Heat Produced by the Electric Current.—The heat in B.t.u. produced by an electric current flowing through a resistance is equal to the square of the cur-

rent in amperes times the resistance of the circuit in ohms times the time in sec. during which the current flows and times .00095 or

Heat in B.t.u. = (the current in amperes)^2 $\times$ resistance in ohms $\times$ seconds $\times$ .00095

Example.—How much heat is produced by a current of 10 amperes flowing for 15 min. through a resistance of 22 ohms?

B.t.u. = $10 \times 10 \times 22 \times 15 \times 60 \times .00095$ or 1881. *Ans.*

Working the other way round we can find the current required to produce a given amount of heat when flowing for a given time through a given resistance.

If H is the heat in B.t.u., I the current in amperes, R the resistance in ohms, and T the time in sec., the rule given above may be written,

$$H = I^2 \times R \times T \times .00095.$$

To find the current "I" in terms of the other quantities we have

$$I^2 = \frac{H}{R \times T \times .00095}$$

and therefore $$I = \sqrt{\frac{H}{R \times T \times .00095}}$$

Example.—What current in amperes is required to produce 5000 B.t.u. in 10 min. when flowing through a resistance of 40 ohms?

Solution:

$$I = \sqrt{\frac{5000}{40 \times 10 \times 60 \times .00095}} \text{ or } \sqrt{\frac{5000}{24{,}000 \times .00095}}$$

This equals $\sqrt{\frac{5000}{22.8}}$ or 14.8 amperes.

To find "R" from the above rule in terms of the other quantities we have

$$R = \frac{H}{I^2 \times T \times .00095}$$

Similarly
$$T = \frac{H}{I^2 \times R \times .00095}$$

PROBLEMS

9. A Centigrade thermometer in a boiler room reads 40° what is the corresponding temperature Fahrenheit?

10. The difference between the temperature of two objects is 20 Fahr. degrees. Express this difference in Cent. degrees.

11. What effect would there be in the readings of a mercury thermometer if the tube were not of uniform bore?

12. A Fahrenheit thermometer gives the temperature of water as 95°. What is the corresponding temperature in degrees Cent.?

13. How much heat is required to raise 100 lb. of water 10° Fahr.?

14. If 10 cu. ft. of water change in temperature from 90° Fahr. to 75° Fahr., how many heat units are liberated?

15. It is desired to raise 5000 lb. of water from 60° Fahr. to 90° Fahr. and supposing 10 per cent. of the heat value of the coal to be used, how much coal is required if there are 12,000 B.t.u. per lb.?

16. How much heat is required to raise 100 lb. of iron from 70° Fahr. to 350° Fahr.? To how many foot-pounds of energy does this correspond?

17. Which requires more energy to raise the temperature of 10 lb. of iron from 0° to 100° Fahr. or to lift 5 tons of iron 1 ft. vertically?

18. If 1 lb. of coal contains 12,000 B.t.u. to what height could a 1200-ton weight be raised by utilizing all the heat value of the coal?

19. An engine raises a casting weighing 1½ tons through a vertical distance of 5 ft. What is the number of B.t.u. which this amount of work represents?

20. How many B.t.u. are required to raise 50 lb. of lead from 72° Fahr. to 520° Fahr.?

21. A car weighing 15 tons is lifted by a crane through a vertical distance of 10 ft. Find the number of foot-pounds of work done and the number of B.t.u. which the work represents.

22. A certain coal gives 14,000 B.t.u. per lb. To what height would 5 lb. of this coal lift a weight of 8 tons if 40 per cent. of the B.t.u. were utilized?

23. If 2 tons of water are used per hr. in an ice plant in freezing water from an average of 70° Fahr., how many B.t.u. are extracted from this water per day of 10 hr.?

24. Which requires the more energy, to raise the temperature of 20 lb. of lead 100° Fahr. or to lift 4 tons of iron 12 ft. vertically?

25. If 1 lb. of coal contains 12,000 B.t.u. to what height could a 10-ton weight be lifted, if 5 per cent. of the heat value of the coal were utilized?

26. A wire cage weighing $1\frac{1}{2}$ tons is lifted 900 ft. four times an hr. How many foot-pounds of work are done on the cage per hr.?

27. If 3 per cent. of the heat value of a 12,000 B.t.u. coal were utilized, how many pounds of coal would be used per hr. in lifting the cage in Prob. 26?

28. Find the heat produced by a current of 20 amperes flowing for 30 min. through a resistance of 24 ohms.

29. How long will it take a current of 10 amperes to produce 6000 B.t.u. when flowing through a resistance of 60 ohms?

30. What resistance is necessary in order to develop 4800 B.t.u. in 30 min. with a current of 8 amperes?

CHAPTER XVI

COMMON OR BRIGG'S LOGARITHMS

48. Definitions.—It is a good thing to become familiar with the use of logarithmic tables and common rules for using the logarithms of numbers.

In working out problems with logarithms we make use of a table of logarithms from which we can get the logarithm of any number of three, four or five figures, depending on how many figures the tables are made to give.

A logarithm of a number is expressed in two parts, one called the *characteristic* and the other the *mantissa*. The *characteristic* is obtained directly from the number and in the logarithm is the figure to the left of the decimal point. The *mantissa* is found in the tables and is the figure to the right of the decimal point in the logarithm.

49. The Characteristic.—When the number whose logarithm we are finding is *greater than one* the characteristic is positive, or plus. When the number is *less than one* the characteristic is *negative*, or minus. The characteristic is determined from the number of figures which make up the number whose logarithm we are finding, and is always *one less than the total number of figures to the left of the decimal point*. For instance, the characteristic of the logarithm for the number 3354 is 3 because there are four figures in the number. The characteristic of the logarithm for the number 384 is 2, for 24.43 it is 1, and for 7 is 0. There are all positive characteristics as the numbers were all greater than 1.

TABLE 1.—COMMON LOGARITHMS

N.	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
N.	0	1	2	3	4	5	6	7	8	9

TABLE 1.—COMMON LOGARITHMS.—*Continued*

N.	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
N.	0	1	2	3	4	5	6	7	8	9

The negative characteristics for numbers *less than one* are determined in the same manner but they are expressed differently. Thus, the characteristic of the logarithm for the number .562 is -1 (minus one) and instead of writing -1 for the characteristic it is customary to change its form by adding 10, or a multiple of 10 to the character istic, and then indicating the subtraction of the same number from this result. Thus a -1 characteristic is written 9. (mantissa) -10. For $-10 + 9 = -1$. The characteristic for the logarithm of the number .062 is -2 and is written 8. (mantissa) -10, since $-10 +$ ° -2.

50. The Mantissa.—The mantissa consists c figures obtained from the tables and compri portion of the logarithm to the right of th point or the decimal part of the logarithm an found by the use of the tables.

The total logarithm for a number is obtained by combining the mantissa read in the tables, with the proper characteristic.

51. Use of the Tables.—The tables give the mantissa of the number only, and are arranged to read the first two figures of the number of which we wish to find the logarithm under the column headed N. and the third figure of the number to the *right* of N. at the top of the table. For example, if we wish to find the logarithm of the number 215, look down the vertical column under N. for the first two figures of the number or 21 and then across the page horizontally to the vertical column headed with the third figure of the number or 5 and read the figures 3324 which is the *mantissa* of the logarithm for the number 215.

The characteristic for 215 is 2. Hence the logarithm of 215 is 2.3324.

The logarithm of the number 384 is 2.5843. (2 is the characteristic) (.5843 is the mantissa.)

The logarithm of the number .287 is -1.4579 (-1 is the characteristic) (.4579 is the mantissa).

The logarithm of the number 26 is 1.4150. (1 is the characteristic and .4150 is the mantissa.)

The logarithm of the number .236 is $9.3729 - 10$. -1 or $(9 - 10)$ is the characteristic and .3729 is the mantissa.

The logarithm of the number .0867 is $8.9380 - 10$. -2 or $(8 - 10)$ is the characteristic and .9380 is the mantissa.

The logarithm of 8.3 is 0.9191. 0 is the characteristic and 0.9191 is the mantissa.

52. To Find a Number Corresponding to a Logarithm. —We find the nearest logarithm to the one given and look across horizontally under column N. for the first two figures of the number and to the top of the column in which the logarithm occurs for the third figure. The characteristic will determine the number of figures to the left of the decimal point in the number. Thus: Find the number corresponding to the logarithm 2.3366. The nearest mantissa is .3365 occurring in the horizontal line opposite the number 21 in column headed N. and in the vertical column headed 7. Hence the number is 217. with the decimal point after the seven because the characteristic is 2 (the number of figures to the left of the decimal point in the number is *one more* than the characteristic).

There are four (4) general rules for the use of logarithms. These may be stated as follows:

Rule 1.—To multiply two numbers:

Add their logarithms and find the number in the table corresponding to the sum of their logarithms. Thus:

Multiply 261 by 885

$$\begin{aligned} \log 261 &= 2.4166 \\ \log 885 &= \underline{2.9469} \\ \text{Sum of logs} &= 5.3635 \\ \text{Number} &= 231000. \end{aligned} \qquad \textit{Ans.}$$

Pointing off one more figure in the number than the characteristic we have six figures before the decimal point in the answer.

PROBLEMS FOR PRACTICE

1. Multiply 235 by 360.
2. Multiply 898 by 210.
3. Multiply 635 by 359.
4. Multiply 110 by 236.
5. Multiply 2.56 by 304.
6. Multiply .279 by 56.3.
7. Multiply .076 by .005.
8. Multiply 9.15 by 12.4.
9. Multiply .028 by .144.
10. Multiply 186 by .062.

Rule 2.—To divide two numbers:

Subtract the logarithm of the subtrahend from the logarithm of the minuend and find the number in the table corresponding to the difference of these logs. Thus:

Divide 836 by 2.10

$$\begin{aligned} \log 836 &= 2.9222 \\ \log 2.10 &= 0.3222 \\ \text{Difference of logs} &= 2.6000 \end{aligned} \qquad \text{Number} = 398. \ \textit{Ans.}$$

Since the characteristic is 2.6000 we point off there figures in the number, the decimal point being placed after the eight.

EXAMPLES FOR PRACTICE

11. Divide 600 by 42.
12. Divide 489 by 5.
13. Divide 628 by 124.
14. Divide 989 by 2.5.
15. Divide 9.67 by .002
16. Divide 1.04 by .02.
17. Divide 98.1 by 12.5.
18. Divide .126 by 6.26.
19. Divide .166 by 1.25.
20. Divide 7.89 by 6.32.

53. To Raise a Number to a Power.—Multiply the logarithm of the number by the exponent or power to which the number is to be raised, and find the number in the table corresponding to the product thus obtained.

Example.—Find the value of $(384)^{2.5}$.

Solution: $\text{Log } 384 = 2.5843.$

$2.5 \times 2.5843 = 6.4608$. The number corresponding to the logarithm $6.4608 = 2890000$. *Ans.* The nearest number corresponding to the mantissa .4608 is 289. The characteristic 6 gives seven figures to the left of the decimal point.

Example.—Find the value of $(.0325)^3$.

Solution:
$$\text{Log } .0325 = 8.5119 - 10$$
$$3 \times (8.5119 - 10) = 25.5357 - 30$$
or
$$-5.5357$$

From the tables the number nearest 5357 is 343.
With a characteristic of -5 the answer is .0000343.

Example.—Find the value of $(.305)^7$.

Solution:
$$\text{Log. } .305 = 9.4843 - 10$$
$$7 \times (9.4843 - 10) = 66.3901 - 70$$
or
$$-4.3901$$

From the tables the number nearest 3901 is 246.
With a characteristic of -4 the answer is .000246.

PROBLEMS FOR PRACTICE

Find the value of

21. $(628)^{3}$ **26.** $(76.3)^{.02}$
22. $(988)^{.2}$ **27.** $(2.8)^{2.5}$
23. $(625)^{7.8}$ **28.** $(.002)^{2.18}$
24. $(888)^{1/8}$ **29.** $(6.25)^{9.02}$
25. $(.867)^{2.62}$ **30.** $(0.0652)^{2.89}$

54. To Find a Root of a Number.

Divide the logarithm of the number by the index of root and find the number corresponding to the quont thus obtained.

Example.—Find the value of $\sqrt[3]{286}$.

Solution: $\text{Log } 286 = 2.4564$

$$\frac{2.4564}{3} = 0.8188$$

From the tables the number nearest .8188 is 659. With a characteristic of 0, the answer is 6.59 that is point off one figure in the number thus bringing the imal point after the six.

Example.—Find the value of $\sqrt[6]{.000625}$.

Solution: $\text{Log } .000625 = 6.7959 - 10$

Before dividing the logarithm by 6, we add and subet from the characteristic a number which is a multiple 10 and which will make the negative portion of the racteristic divisible by six without a remainder. If add and subtract 50 the logarithm $6.7959 - 10$ beles $56.7959 - 60$.

Then $$\frac{56.7959 - 60}{6} = 9.46598 - 10$$

$$9.4660 - 10$$
$$-1.4660$$

rom the tables the number nearest 4660 is 292. With a characteristic of -1 the answer is .292.

EXAMPLES FOR PRACTICE

31. $\sqrt{102}$

32. $\sqrt[3]{3.85}$

33. $\sqrt[6]{360}$

34. $\sqrt[7]{206}$

35. $\sqrt[0.4]{.0265}$

36. $\sqrt[6]{.260}$

37. $\sqrt[1.1]{4.26}$

38. $\sqrt[9]{1.02}$

39. $\sqrt[3]{12.6}$

40. $\sqrt[5]{32.6}$

MISCELLANEOUS PROBLEMS FOR PRACTICE

(To be solved by logarithms)

41. Find the circumferences of circles whose diameters are as follows: 126 ft. 0 in. 7 ft. 6¾ in. 6 ft. 11.2 in. 1720 ft. 0 in. 7890 ft. 0 in. (Circumference = $3.14 \times D$ where D = diameter.)

42. What are the areas in sq. in. of circles with the following diameters: 62.2 in. 180 in. 9 ft. 6 in. 7 ft. 11$\frac{5}{16}$ in. 5 ft. 10½ in. (Area = $.785D^2$ where D = diameter.)

43. What are the diameters in in. of circles whose areas are: 6450 sq. in.? 5010 sq. in.? 4050 sq. in.? 7370 sq. in ?

$$\left(D = \sqrt{\frac{\text{area}}{0.785}} \text{ or } D = \frac{1}{0.887}\sqrt{\text{area}} \text{ where } D = \text{diameter.}\right)$$

44. Find the horsepower of a locomotive from the following rule:

$$\text{Horsepower} = \frac{PLAN}{33{,}000} = \frac{130 \times 28 \times 24 \times 24 \times .785 \times 300 \times 2}{12 \times 33{,}000}$$

in which P = 130 lb. per sq. in. pressure on the piston, cylinders 24 in. $\times$ 28 in., N = 300 strokes per min.

Work out the following problems by logarithms, showing all the work:

45. $\dfrac{43.8 \times 2760 \times .97 \times 3.14}{7.85 \times 2.33 \times 42.6 \times .785}$

46. $\sqrt{294}$. $\sqrt{4.84}$. $\sqrt[3]{3270}$. $\sqrt[3]{8.24}$.

47. $(29.7)^2$. $(436)^3$. $(2.86)^2$. $(6.78)^3$.

48. Find the horsepower of a Pacific type locomotive with cylinders 24 in. $\times$ 26 in., drivers 80 in., running at 40 m.p.h., and ef-

ctive steam pressure of 160 lb. per sq. in. using the following gures:

$$\text{Horsepower} = \frac{PLAN}{33{,}000}$$

$$\frac{160 \times 26 \times 24 \times 24 \times .785 \times 40 \times 5280 \times 12 \times 2 \times 2}{12 \times 33{,}000 \times 80 \times 3.14 \times 60}$$

49. An axle weighs originally 1050 lb. It loses $5\frac{3}{8}$ per cent. of its eight when turned down. Find the weight of the material re-oved, the number of cubic inches of material removed, and the ıal weight of the axle. The material weighs .28 lb. per cu. in.

50. Find the capacity in gal. and in cu. ft. of a cylindrical tank : ft. high and 10 ft., in diam.

51. Design a cylindrical tank to hold 20,000 gal.

52. Design a coal bin to hold 300 tons of coal (one ton oc-.pies about 37 cu. ft.).

53. How many cylindrically shaped iron castings 4 in. diam. ıd $1\frac{1}{2}$ ft. long can be made from 10 tons of metal. (Iron weighs 6 lb. per cu. in.)

CHAPTER XVII

THE MEASUREMENT OF RIGHT TRIANGLES

55. Definition.—In the work which follows are rules for finding the sides and angles of right triangles, that is, those having a 90 degree (90°) or right angle. These rules are used to work out problems in the shop work as shown by examples which follow.

The rules for working out triangles (that is, figures with three angles and also three sides) form a branch of mathematics called "trigonometry," "tri" meaning

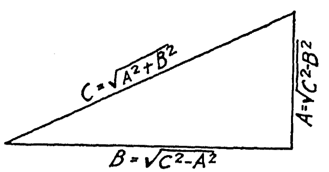


FIG. 104.

three, "gono" meaning *side* and "metry" *measurement*, or the measurement of three-sided figures. We need not think of the work necessarily as "trigonometry," but principally how it applies in working out problems that cannot be done by rules we have had before. We can use these rules also to do some of the problems quicker than by rules we have had up to this time.

56. Right Triangle Rule.—We have already had the rule that in a right triangle "the square of the hypothe-

nuse is equal to the sum of the squares of the other two sides," that is, referring to Fig. 104, if the hypothenuse is C, the height A, and the base B then

$$(\text{hyp.})^2 = (\text{height})^2 + (\text{base})^2$$

that is $C^2 = A^2 + B^2$ from this

$$C = \sqrt{A^2 + B^2}$$

also since $C^2 = A^2 + B^2$

$$A^2 = C^2 - B^2$$

and $A = \sqrt{C^2 - B^2}$

and also $B = \sqrt{C^2 - A^2}$

That is when we have any two sides of a right triangle we can find the other side from one of these rules. *It often is necessary,* however, to find a side of a right triangle when we know only one other side and only one acute angle of the triangle. For example, in Fig. 105

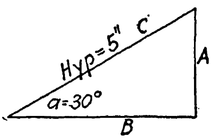


FIG. 105.

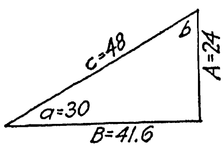


FIG. 106.

the hypothenuse is 5 in. and the angle $a = 30°$. If we want to find either side A or B we cannot use the rules $A = \sqrt{C^2 - B^2}$ and $B = \sqrt{C^2 - A^2}$ since we know only one side in either of these equations. To find either side A or B we use certain *ratios* between the sides of the right triangle. These ratios have already been worked out for us in tables for different angles in the triangle.

If as in Fig. 106, angle $a = 30°$. Side $A = 24$ and side $C = 48$.

$$\frac{\text{Side } A}{\text{Side } C} = \frac{\text{side opposite angle } a}{\text{hypothenuse}} = \frac{24}{48} = \frac{1}{2} = .5$$

57. Definition of Sine, Cosine, Tangent, and Cotangent.—This *ratio* of the opposite side and the hypothenuse is called the *sine* of angle a and for 30° it is .5. Similar ratios are worked out for all angles between 0° and 90° and put into tables for our use. If we divide the base B by the hypothenuse we have (adjacent meaning the side forming one side of the angle)

$$\frac{\text{side } B}{\text{side } C} = \frac{\text{side adjacent to angle } a}{\text{hypothenuse}} = \frac{41.6}{48} = .866.$$

This ratio is called the *cosine* of angle a. Similar ratios are worked out for all angles and put into tables for our use.
If we divide side A by side B we have

$$\frac{\text{side } A}{\text{side } B} = \frac{\text{side opposite angle } a}{\text{side adjacent to angle } a} = \frac{24}{41.6} = .577$$

This is called the *tangent* of angle a. Also

$$\frac{\text{side } B}{\text{side } A} = \frac{\text{side adjacent to angle } a}{\text{side opposite angle } a} = \frac{41.6}{24} = 1.73$$

This is called the *cotangent* of angle a.

These four ratios are what we will use in working out our triangles. They are called *functions* of the angle considered. Put in tabular form these functions are:

58. Tables of Rules for Functions of an Angle

sin	$= \dfrac{\text{opposite side}}{\text{hyp.}}$	: side opp. = sin × hyp.: hyp. $= \dfrac{\text{side opp.}}{\text{sin}}$
cos	$= \dfrac{\text{adjacent side}}{\text{hyp.}}$	: side adj. = cos × hyp.: hyp. $= \dfrac{\text{side adj.}}{\text{cos}}$
tan	$= \dfrac{\text{opposite side}}{\text{adjacent side}}$	: side opp. = tan × side adj.: side adj. $= \dfrac{\text{side opp.}}{\text{tan}}$
cot	$= \dfrac{\text{adjacent side}}{\text{opposite side}}$	: side adj. = cot × side opp.: side opp. $= \dfrac{\text{side adj.}}{\text{cot}}$

opp. = opposite
hyp. = hypothenuse
adj. = adjacent

59. Calculation of Functions for 30° and 45°.—Referring to Fig. 107, in which angle $a = 30°$, angle $b = 60°$, side $A = 10$, $B = 17.3$, and $C = 20$.

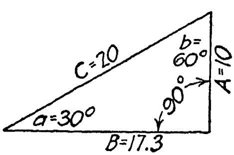


FIG. 107.

$$\sin a = \sin 30° = \frac{\text{opp. side}}{\text{hyp.}} = \frac{10}{20} = .5$$

$$\cos a = \cos. 30° = \frac{\text{adj. side}}{\text{hyp.}} = \frac{17.3}{20} = .866$$

$$\tan a = \tan 30° = \frac{\text{opp. side}}{\text{adj. side}} = \frac{10}{17.3} = .577$$

$$\cot a = \cot 30° = \frac{\text{adj. side}}{\text{opp. side}} = \frac{17.3}{10} = 1.73$$

the same way for Fig. 108, in which $a = 45°$, $b =$
$A = 20$, $B = 20$, and $C = 28.3$

$$\sin a = \sin 45° = \frac{\text{opp. side}}{\text{hyp.}} = \frac{20}{28.3} = .707$$

$$\cos a = \cos 45° = \frac{\text{adj. side}}{\text{hyp.}} = \frac{20}{28.3} = .707$$

$$\tan a = \tan 45° = \frac{\text{opp. side}}{\text{adj. side}} = \frac{20}{20} = 1.00$$

$$\cot a = \cot 45° = \frac{\text{adj. side}}{\text{opp. side}} = \frac{20}{20} = 1.00$$

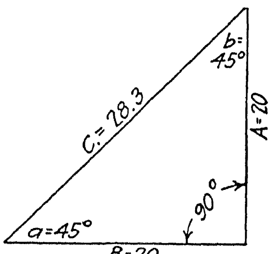


FIG. 108.

Explanation of Use of Tables.—These functions, ios of sides for a right triangle can be calculated for iven angle by constructing the right triangle and ıring the length of its sides, but tables are prepared give us these ratios for all angles from 0° to 90° ıute steps (see pages 133 to 156). These are the which we use in making calculations. In using tables the angle in degrees is read in the horizontal ; the top of the page for angles up to 44° and the es in the vertical columns to the left. Angles of over are read in the horizontal line at the bottom page for degrees and the minutes in the verti-

cal columns to the right. Thus page 134 gives by minute steps, the *sines* and *cosines* of all angles, from 1° to 4° inclusive and the *sines* and *cosines* of all angles from 85° to 88° inclusive. Page 135 gives by minute steps the *sines* and *cosines* of all angles from 5° to 8° inclusive and the *sines cosines* of all angles from 81° to 84° inclusive. Page 145 gives the *tangents* and *cotangents* of angles from 0° to 3° and the *tangents* and *contangents* of angles from 86° to 89° and page 146 gives the *tangents* and *cotangents* of angles from 4° to 7° and *tangents* and *cotangents* of angles from 82° to 85°. Referring to page 137.

The sine of 14° 30′ = .25038
The sine of 13° 20′ = .23062
The sine of 16° 40′ = .28680

From page 143 the cosine of 50° 20′ = .63832 (use the right-hand column of minutes and degrees at the *bottom* of the page).

The cosine of 73° 50′ = .27843. The cosine of 0° 15′ = .99999. The tables of tangents and cotangents, pages 145 to 146 are read in the same way.

PROBLEMS

1. From the tables of sines and cosines find the *sines* of the following angles: 15°, 30°, 60°, 90° (that is 89° 60′), 32° 10′, 43° 42′, 2° 4′, 44° 50′, 40° 55′, 45°.

2. Find the cosines of the same angles given in Prob. 1.

3. From the table of tangents and contangents find the *tangents* of the following angles: 14° 10′, 27° 57′, 42° 18′, 53° 12′, 63′, 28°, 45°, 30°, 17° 27′, 36°.

4. Find the cotangents of the same angles given in Prob. 3.

5. Using the tables of functions complete the following table, finding first the angle from the function given, and then knowing the angle, find the other functions.

Prob. No.	Angle	Sine	Cosine	Tangent	Cotangent
1		.21644			
2			.48226		
3		.82248			
4			.87462		
5				6.31375	
6					1.12369

61. Tables of Natural Sines, Cosines, Tangents, and Cotangents of Angles from 0° to 90° Varying by Minute Angles.

	0°				0°				0°		
′	Sine	Cosine	′	′	Sine	Cosine	′	′	Sine	Cosine	′
0	.00000	1	60	21	.00611	.99998	39	41	.01193	.99993	19
1	.00029	1	59	22	.00640	.99998	38	42	.01222	.99993	18
2	.00058	1	58	23	.00669	.99998	37	43	.01251	.99992	17
3	.00087	1	57	24	.00698	.99998	36	44	.01280	.99992	16
4	.00116	1	56	25	.00727	.99997	35	45	.01309	.99991	15
5	.00145	1	55	26	.00756	.99997	34	46	.01338	.99991	14
6	.00175	1	54	27	.00785	.99997	33	47	.01367	.99991	13
7	.00204	1	53	28	.00814	.99997	32	48	.01396	.99990	12
8	.00233	1	52	29	.00844	.99996	31	49	.01425	.99990	11
9	.00262	1	51	30	.00873	.99996	30	50	.01454	.99989	10
10	.00291	1	50	31	.00902	.99996	29	51	.01483	.99989	9
11	.00320	.99999	49	32	.00931	.99996	28	52	.01513	.99989	8
12	.00349	.99999	48	33	.00960	.99995	27	53	.01542	.99988	7
13	.00378	.99999	47	34	.00989	.99995	26	54	.01571	.99988	6
14	.00407	.99999	46	35	.01018	.99995	25	55	.01600	.99987	5
15	.00436	.99999	45	36	.01047	.99995	24	56	.01629	.99987	4
16	.00465	.99999	44	37	.01076	.99994	23	57	.01658	.99986	3
17	.00495	.99999	43	38	.01105	.99994	22	58	.01687	.99986	2
18	.00524	.99999	42	39	.01134	.99994	21	59	.01716	.99985	1
19	.00553	.99998	41	40	.01164	.99993	20	60	.01745	.99985	0
20	.00582	.99998	40								
′	Cosine	Sine	′	′	Cosine	Sine	′	′	Cosine	Sine	′
	89°				89°				89°		

	1°		2°		3°		4°		
′	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	′
0	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	88°		87°		86°		85°		

	5°		6°		7°		8°		
′	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	′
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	0
′	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	′
	84°		83°		82°		81°		

′	9° Sine	9° Cosine	10° Sine	10° Cosine	11° Sine	11° Cosine	12° Sine	12° Cosine	′
0	.15643	.98769	.17365	.98481	.19081	.98163	.20791	.97815	60
1	.15672	.98764	.17393	.98476	.19109	.98157	.20820	.97809	59
2	.15701	.98760	.17422	.98471	.19138	.98152	.20848	.97803	58
3	.15730	.98755	.17451	.98466	.19167	.98146	.20877	.97797	57
4	.15758	.98751	.17479	.98461	.19195	.98140	.20905	.97791	56
5	.15787	.98746	.17508	.98455	.19224	.98135	.20933	.97784	55
6	.15816	.98741	.17537	.98450	.19252	.98129	.20962	.97778	54
7	.15845	.98737	.17565	.98445	.19281	.98124	.20990	.97772	53
8	.15873	.98732	.17594	.98440	.19309	.98118	.21019	.97766	52
9	.15902	.98728	.17623	.98435	.19338	.98112	.21047	.97760	51
10	.15931	.98723	.17651	.98430	.19366	.98107	.21076	.97754	50
11	.15959	.98718	.17680	.98425	.19395	.98101	.21104	.97748	49
12	.15988	.98714	.17708	.98420	.19423	.98096	.21132	.97742	48
13	.16017	.98709	.17737	.98414	.19452	.98090	.21161	.97735	47
14	.16046	.98704	.17766	.98409	.19481	.98084	.21189	.97729	46
15	.16074	.98700	.17794	.98404	.19509	.98079	.21218	.97723	45
16	.16103	.98695	.17823	.98399	.19538	.98073	.21246	.97717	44
17	.16132	.98690	.17852	.98394	.19566	.98067	.21275	.97711	43
18	.16160	.98689	.17880	.98389	.19595	.98061	.21303	.97705	42
19	.16189	.98681	.17909	.98383	.19623	.98056	.21331	.97698	41
20	.16218	.98676	.17937	.98378	.19652	.98050	.21360	.97692	40
21	.16246	.98671	.17966	.98373	.19680	.98044	.21388	.97686	39
22	.16275	.98667	.17995	.98368	.19709	.98039	.21417	.97680	38
23	.16304	.98662	.18023	.98362	.19737	.98033	.21445	.97673	37
24	.16333	.98657	.18052	.98357	.19766	.98027	.21474	.97667	36
25	.16361	.98652	.18081	.98352	.19794	.98021	.21502	.97661	35
26	.16390	.98648	.18109	.98347	.19823	.98016	.21530	.97655	34
27	.16419	.98643	.18138	.98341	.19851	.98010	.21559	.97648	33
28	.16447	.98638	.18166	.98336	.19880	.98004	.21587	.97642	32
29	.16476	.98633	.18195	.98331	.19908	.97987	.21616	.97636	31
30	.16505	.98629	.18224	.98325	.19937	.97992	.21644	.97630	30
31	.16533	.98624	.18252	.98320	.19965	.97987	.21672	.97623	29
32	.16562	.98619	.18281	.98315	.19994	.97981	.21701	.97617	28
33	.16591	.98614	.18309	.98310	.20022	.97975	.21729	.97611	27
34	.16620	.98609	.18338	.98304	.20031	.97969	.21758	.97604	26
35	.16648	.98604	.18367	.98299	.20079	.97963	.21786	.97598	25
36	.16677	.98600	.18395	.98294	.20108	.97958	.21814	.97592	24
37	.16706	.98595	.18424	.98288	.20136	.97952	.21843	.97585	23
38	.16734	.98590	.18452	.98283	.20165	.97946	.21871	.97579	22
39	.16763	.98585	.18481	.98277	.20193	.97940	.21899	.97573	21
40	.16792	.98580	.18509	.98272	.20222	.97934	.21928	.97566	20
41	.16820	.98575	.18538	.98267	.20250	.97928	.21956	.97560	19
42	.16849	.98570	.18567	.98261	.20279	.97922	.21985	.97553	18
43	.16878	.98565	.18595	.98256	.20307	.97916	.22013	.97547	17
44	.16906	.98561	.18624	.98250	.20336	.97910	.22041	.97541	16
45	.16935	.98556	.18652	.98245	.20364	.97905	.22070	.97534	15
46	.16964	.98551	.18681	.98240	.20393	.97899	.22098	.97528	14
47	.16992	.98546	.18710	.98234	.20421	.97893	.22126	.97521	13
48	.17021	.98541	.18738	.98229	.20450	.97887	.22155	.97515	12
49	.17050	.98536	.18767	.98223	.20478	.97881	.22183	.97508	11
50	.17078	.98531	.18795	.98218	.20507	.97875	.22212	.97502	10
51	.17107	.98526	.18824	.98212	.20535	.97869	.22240	.97496	9
52	.17136	.98521	.18852	.98207	.20563	.97863	.22268	.97489	8
53	.17164	.98516	.18881	.98201	.20592	.97857	.22297	.97483	7
54	.17193	.98511	.18910	.98196	.20620	.97851	.22325	.97476	6
55	.17222	.98506	.18938	.98190	.20649	.97845	.22353	.97470	5
56	.17250	.98501	.18967	.98185	.20677	.97839	.22382	.97463	4
57	.17279	.98496	.18995	.98179	.20706	.97833	.22410	.97457	3
58	.17308	.98491	.19024	.98174	.20734	.97827	.22438	.97450	2
59	.17336	.98486	.19052	.98168	.20763	.97821	.22467	.97444	1
60	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	80°		79°		78°		77°		

′	13°		14°		15°		16°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.22495	.97437	.24192	.97030	.25882	.96593	.27564	.96126	60
1	.22523	.97430	.24220	.97023	.25910	.96585	.27592	.96118	59
2	.22552	.97424	.24249	.97015	.25938	.96578	.27620	.96110	58
3	.22580	.97417	.24277	.97008	.25966	.96570	.27648	.96102	57
4	.22608	.97411	.24305	.97001	.25994	.96562	.27676	.96094	56
5	.22637	.97404	.24333	.96994	.26022	.96555	.27704	.96086	55
6	.22665	.97398	.24362	.96987	.26050	.96547	.27731	.96078	54
7	.22693	.97391	.24390	.96980	.26079	.96540	.27759	.96070	53
8	.22722	.97384	.24418	.96973	.26107	.96532	.27787	.96062	52
9	.22750	.97378	.24446	.96966	.26135	.96524	.27815	.96054	51
10	.22778	.97371	.24474	.96959	.26163	.96517	.27843	.96046	50
11	.22807	.97365	.24503	.96952	.26191	.96509	.27871	.96037	49
12	.22835	.97358	.24531	.96945	.26219	.96502	.27899	.96029	48
13	.22863	.97351	.24559	.96937	.26247	.96494	.27927	.96021	47
14	.22892	.97345	.24587	.96930	.26275	.96486	.27955	.96013	46
15	.22920	.97338	.24615	.96923	.26303	.96479	.27983	.96005	45
16	.22948	.97331	.24644	.96916	.26331	.96471	.28011	.95997	44
17	.22977	.97325	.24672	.96909	.26359	.96463	.28039	.95989	43
18	.23005	.97318	.24700	.96902	.26387	.96456	.28067	.95981	42
19	.23033	.97311	.24728	.96894	.26415	.96448	.28095	.95972	41
20	.23062	.97304	.24756	.96887	.26443	.96440	.28123	.95964	40
21	.23090	.97298	.24784	.96880	.26471	.96433	.28150	.95956	39
22	.23118	.97291	.24813	.96873	.26500	.96425	.28178	.95948	38
23	.23146	.97284	.24841	.96866	.26528	.96417	.28206	.95940	37
24	.23175	.97278	.24869	.96858	.26556	.96410	.28234	.95931	36
25	.23203	.97271	.24897	.96851	.26584	.96402	.28262	.95923	35
26	.23231	.97264	.24925	.96844	.26612	.96394	.28290	.95915	34
27	.23260	.97257	.24954	.96837	.26640	.96386	.28318	.95907	33
28	.23288	.97251	.24982	.96829	.26668	.96379	.28346	.95898	32
29	.23316	.97244	.25010	.96822	.26696	.96371	.28374	.95890	31
30	.23345	.97237	.25038	.96815	.26724	.96363	.28402	.95882	30
31	.23373	.97230	.25066	.96807	.26752	.96355	.28429	.95874	29
32	.23401	.97223	.25094	.96800	.26780	.96347	.28457	.95865	28
33	.23429	.97217	.25122	.96793	.26808	.96340	.28485	.95857	27
34	.23458	.97210	.25151	.96786	.26836	.96332	.28513	.95849	26
35	.23486	.97203	.25179	.96778	.26864	.96324	.28541	.95841	25
36	.23514	.97196	.25207	.96771	.26892	.96316	.28569	.95832	24
37	.23542	.97189	.25235	.96764	.26920	.96308	.28597	.95824	23
38	.23571	.97182	.25263	.96756	.26948	.96301	.28625	.95816	22
39	.23599	.97176	.25291	.96749	.26976	.96293	.28652	.95807	21
40	.23627	.97169	.25320	.96742	.27004	.96285	.28680	.95799	20
41	.23656	.97162	.25348	.96734	.27032	.96277	.28708	.95791	19
42	.23684	.97155	.25376	.96727	.27060	.96269	.28736	.95782	18
43	.23712	.97148	.25404	.96719	.27088	.96261	.28764	.95774	17
44	.23740	.97141	.25432	.96712	.27116	.96253	.28792	.95766	16
45	.23769	.97134	.25460	.96705	.27144	.96246	.28820	.95757	15
46	.23797	.97127	.25488	.96697	.27172	.96238	.28847	.95749	14
47	.23825	.97120	.25516	.96690	.27200	.96230	.28875	.95740	13
48	.23853	.97113	.25545	.96682	.27228	.96222	.28903	.95732	12
49	.23882	.97106	.25573	.96675	.27256	.96214	.28931	.95724	11
50	.23910	.97100	.25601	.96667	.27284	.96206	.28959	.95715	10
51	.23938	.97093	.25629	.96660	.27312	.96198	.28987	.95707	9
52	.23966	.97086	.25657	.96653	.27340	.96190	.29015	.95698	8
53	.23995	.97079	.25685	.96645	.27368	.96182	.29042	.95690	7
54	.24023	.97072	.25713	.96638	.27396	.96174	.29070	.95681	6
55	.24051	.97065	.25741	.96630	.27424	.96166	.29098	.95673	5
56	.24079	.97058	.25769	.96623	.27452	.96158	.29126	.95664	4
57	.24108	.97051	.25798	.96615	.27480	.96150	.29154	.95656	3
58	.24136	.97044	.25826	.96608	.27508	.96142	.29182	.95647	2
59	.24164	.97037	.25854	.96600	.27536	.96134	.29209	.95639	1
60	.24192	.97030	.25882	.96593	.27564	.96126	.29237	.95630	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	76°		75°		74°		73°		

′	17° Sine	17° Cosine	18° Sine	18° Cosine	19° Sine	19° Cosine	20° Sine	20° Cosine	′
0	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	60
1	.29265	.95622	.30929	.95097	.32584	.94542	.34229	.93959	59
2	.29293	.95613	.30957	.95088	.32612	.94533	.34257	.93949	58
3	.29321	.95605	.30985	.95079	.32639	.94523	.34284	.93939	57
4	.29348	.95596	.31012	.95070	.32667	.94514	.34311	.93929	56
5	.29376	.95588	.31040	.95061	.32694	.94504	.34339	.93919	55
6	.29404	.95579	.31068	.95052	.32722	.94495	.34366	.93909	54
7	.29432	.95571	.31095	.95043	.32749	.94485	.34393	.93899	53
8	.29460	.95562	.31123	.95033	.32777	.94476	.34421	.93889	52
9	.29487	.95554	.31151	.95024	.32804	.94466	.34448	.93879	51
10	.29515	.95545	.31178	.95015	.32832	.94457	.34475	.93869	50
11	.29543	.95536	.31206	.95006	.32859	.94447	.34503	.93859	49
12	.29571	.95528	.31233	.94997	.32887	.94438	.34530	.93849	48
13	.29599	.95519	.31261	.94988	.32914	.94428	.34557	.93839	47
14	.29626	.95511	.31289	.94979	.32942	.94418	.34584	.93829	46
15	.29654	.95502	.31316	.94970	.32969	.94409	.34612	.93819	45
16	.29682	.95493	.31344	.94961	.32997	.94399	.34639	.93809	44
17	.29710	.95485	.31372	.94952	.33024	.94390	.34666	.93799	43
18	.29737	.95476	.31399	.94943	.33051	.94380	.34694	.93789	42
19	.29765	.95467	.31427	.94933	.33079	.94370	.34721	.93779	41
20	.29793	.95459	.31454	.94924	.33106	.94361	.34748	.93769	40
21	.29821	.95450	.31482	.94915	.33134	.94351	.34775	.93759	39
22	.29849	.95441	.31510	.94906	.33161	.94342	.34803	.93748	38
23	.29876	.95433	.31537	.94897	.33189	.94332	.34830	.93738	37
24	.29904	.95424	.31565	.94888	.33216	.94322	.34857	.93728	36
25	.29932	.95415	.31593	.94878	.33244	.94313	.34884	.93718	35
26	.29960	.95407	.31620	.94869	.33271	.94303	.34912	.93708	34
27	.29987	.95398	.31648	.94860	.33298	.94293	.34939	.93698	33
28	.30015	.95389	.31675	.94851	.33326	.94284	.34966	.93688	32
29	.30043	.95380	.31703	.94842	.33353	.94274	.34993	.93677	31
30	.30071	.95372	.31730	.94832	.33381	.94264	.35021	.93667	30
31	.30098	.95363	.31758	.94823	.33408	.94254	.35048	.93657	29
32	.30126	.95354	.31786	.94814	.33436	.94245	.35075	.93647	28
33	.30154	.95345	.31813	.94805	.33463	.94235	.35102	.93637	27
34	.30182	.95337	.31841	.94795	.33490	.94225	.35130	.93626	26
35	.30209	.95328	.31868	.94786	.33518	.94215	.35157	.93616	25
36	.30237	.95319	.31896	.94777	.33545	.94206	.35184	.93606	24
37	.30265	.95310	.31923	.94768	.33573	.94196	.35211	.93596	23
38	.30292	.95301	.31951	.94758	.33600	.94186	.35239	.93585	22
39	.30320	.95293	.31979	.94749	.33627	.94176	.35266	.93575	21
40	.30348	.95284	.32006	.94740	.33655	.94167	.35293	.93565	20
41	.30376	.95275	.32034	.94730	.33682	.94157	.35320	.93555	19
42	.30403	.95266	.32061	.94721	.33710	.94147	.35347	.93544	18
43	.30431	.95257	.32089	.94712	.33737	.94137	.35375	.93534	17
44	.30459	.95248	.32116	.94702	.33764	.94127	.35402	.93524	16
45	.30486	.95240	.32144	.94693	.33792	.94118	.35429	.93514	15
46	.30514	.95231	.32171	.94684	.33819	.94108	.35456	.93503	14
47	.30542	.95222	.32199	.94674	.33846	.94098	.35484	.93493	13
48	.30570	.95213	.32227	.94665	.33874	.94088	.35511	.93483	12
49	.30597	.95204	.32254	.94656	.33901	.94078	.35538	.93472	11
50	.30625	.95195	.32282	.94646	.33929	.94068	.35565	.93462	10
51	.30653	.95186	.32309	.94637	.33956	.94058	.35592	.93452	9
52	.30680	.95177	.32337	.94627	.33983	.94049	.35619	.93441	8
53	.30708	.95168	.32364	.94618	.34011	.94039	.35647	.93431	7
54	.30736	.95159	.32392	.94609	.34038	.94029	.35674	.93420	6
55	.30763	.95150	.32419	.94599	.34065	.94019	.35701	.93410	5
56	.30791	.95142	.32447	.94590	.34093	.94009	.35728	.93400	4
57	.30819	.95133	.32474	.94580	.34120	.93999	.35755	.93389	3
58	.30846	.95124	.32502	.94571	.34147	.93989	.35782	.93379	2
59	.30874	.95115	.32529	.94561	.34175	.93979	.35810	.93368	1
60	.30902	.95106	.32557	.94552	.34202	.93969	.35837	.93358	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	72°		71°		70°		69°		

′	21°		22°		23°		24°		′
	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	
0	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	COSINE	SINE	COSINE	SINE	COSINE	SINE	COSINE	SINE	′
	68°		67°		66°		65°		

′	25° Sine	25° Cosine	26° Sine	26° Cosine	27° Sine	27° Cosine	28° Sine	28° Cosine	′
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	9
52	.43628	.89981	45192	.89206	.46742	.88404	.48277	.87575	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	64°		63°		62°		61°		

′	29° Sine	29° Cosine	30° Sine	30° Cosine	31° Sine	31° Cosine	32° Sine	32° Cosine	′
0	.48481	.87462	.50000	.86603	.51504	.85717	.52992	.84805	60
1	.48506	.87448	.50025	.86588	.51529	.85702	.53017	.84789	59
2	.48532	.87434	.50050	.86573	.51554	.85687	.53041	.84774	58
3	.48557	.87420	.50076	.86559	.51579	.85672	.53066	.84759	57
4	.48583	.87406	.50101	.86544	.51604	.85657	.53091	.84743	56
5	.48608	.87391	.50126	.86530	.51628	.85642	.53115	.84728	55
6	.48634	.87377	.50151	.86515	.51653	.85627	.53140	.84712	54
7	.48659	.87363	.50176	.86501	.51678	.85612	.53164	.84697	53
8	.48684	.87349	.50201	.86486	.51703	.85597	.53189	.84681	52
9	.48710	.87335	.50227	.86471	.51728	.85582	.53214	.84666	51
10	.48735	.87321	.50252	.86457	.51753	.85567	.53238	.84650	50
11	.48761	.87306	.50277	.86442	.51778	.85551	.53263	.84635	49
12	.48786	.87292	.50302	.86427	.51803	.85536	.53288	.84619	48
13	.48811	.87278	.50327	.86413	.51828	.85521	.53312	.84604	47
14	.48837	.87264	.50352	.86398	.51852	.85506	.53337	.84588	46
15	.48862	.87250	.50377	.86384	.51877	.85491	.53361	.84573	45
16	.48888	.87235	.50403	.86369	.51902	.85476	.53386	.84557	44
17	.48913	.87221	.50428	.86354	.51927	.85461	.53411	.84542	43
18	.48938	.87207	.50453	.86340	.51952	.85446	.53435	.84526	42
19	.48964	.87193	.50478	.86325	.51977	.85431	.53460	.84511	41
20	.48989	.87178	.50503	.86310	.52002	.85416	.53484	.84495	40
21	.49014	.87164	.50528	.86295	.52026	.85401	.53509	.84480	39
22	.49040	.87150	.50553	.86281	.52051	.85385	.53534	.84464	38
23	.49065	.87136	.50578	.86266	.52076	.85370	.53558	.84448	37
24	.49090	.87121	.50603	.86251	.52101	.85355	.53583	.84433	36
25	.49116	.87107	.50628	.86237	.52126	.85340	.53607	.84417	35
26	.49141	.87093	.50654	.86222	.52151	.85325	.53632	.84402	34
27	.49166	.87079	.50679	.86207	.52175	.85310	.53656	.84386	33
28	.49192	.87064	.50704	.86192	.52200	.85294	.53681	.84370	32
29	.49217	.87050	.50729	.86178	.52225	.85279	.53705	.84355	31
30	.49242	.87036	.50754	.86163	.52250	.85264	.53730	.84339	30
31	.49268	.87021	.50779	.86148	.52275	.85249	.53754	.84324	29
32	.49293	.87007	.50804	.86133	.52299	.85234	.53779	.84308	28
33	.49318	.86993	.50829	.86119	.52324	.85218	.53804	.84292	27
34	.49344	.86978	.50854	.86104	.52349	.85203	.53828	.84277	26
35	.49369	.86964	.50879	.86089	.52374	.85188	.53853	.84261	25
36	.49394	.86949	.50904	.86074	.52399	.85173	.53877	.84245	24
37	.49419	.86935	.50929	.86059	.52423	.85157	.53902	.84230	23
38	.49445	.86921	.50954	.86045	.52448	.85142	.53926	.84214	22
39	.49470	.86906	.50979	.86030	.52473	.85127	.53951	.84198	21
40	.49495	.86892	.51004	.86015	.52498	.85112	.53975	.84182	20
41	.49521	.86878	.51029	.86000	.52522	.85096	.54000	.84167	19
42	.49546	.86863	.51054	.85985	.52547	.85081	.54024	.84151	18
43	.49571	.86849	.51079	.85970	.52572	.85066	.54049	.84135	17
44	.49596	.86834	.51104	.85956	.52597	.85051	.54073	.84120	16
45	.49622	.86820	.51129	.85941	.52621	.85035	.54097	.84104	15
46	.49647	.86805	.51154	.85926	.52646	.85020	.54122	.84088	14
47	.49672	.86791	.51179	.85911	.52671	.85005	.54146	.84072	13
48	.49697	.86777	.51204	.85896	.52696	.84989	.54171	.84057	12
49	.49723	.86762	.51229	.85881	.52720	.84974	.54195	.84041	11
50	.49748	.86748	.51254	.85866	.52745	.84959	.54220	.84025	10
51	.49773	.86733	.51279	.85851	.52770	.84943	.54244	.84009	9
52	.49798	.86719	.51304	.85836	.52794	.84928	.54269	.83994	8
53	.49824	.86704	.51329	.85821	.52819	.84913	.54293	.83978	7
54	.49849	.86690	.51354	.85806	.52844	.84897	.54317	.83962	6
55	.49874	.86675	.51379	.85792	.52869	.84882	.54342	.83946	5
56	.49899	.86661	.51404	.85777	.52893	.84866	.54366	.83930	4
57	.49924	.86646	.51429	.85762	.52918	.84851	.54391	.83915	3
58	.49950	.86632	.51454	.85747	.52943	.84836	.54415	.83899	2
59	.49975	.86617	.51479	.85732	.52967	.84820	.54440	.83883	1
60	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	0
′	Cosine 60°	Sine 60°	Cosine 59°	Sine 59°	Cosine 58°	Sine 58°	Cosine 57°	Sine 57°	′

′	33° Sine	33° Cosine	34° Sine	34° Cosine	35° Sine	35° Cosine	36° Sine	36° Cosine	′
0	.54464	.83867	.55919	.82904	.57358	.81915	.58779	.80902	60
1	.54488	.83851	.55943	.82887	.57381	.81899	.58802	.80885	59
2	.54513	.83835	.55968	.82871	.57405	.81882	.58826	.80867	58
3	.54537	.83819	.55992	.82855	.57429	.81865	.58849	.80850	57
4	.54561	.83804	.56016	.82839	.57453	.81848	.58873	.80833	56
5	.54586	.83788	.56040	.82822	.57477	.81832	.58896	.80816	55
6	.54610	.83772	.56064	.82806	.57501	.81815	.58920	.80799	54
7	.54635	.83756	.56088	.82790	.57524	.81798	.58943	.80782	53
8	.54659	.83740	.56112	.82773	.57548	.81782	.58967	.80765	52
9	.54683	.83724	.56136	.82757	.57572	.81765	.58990	.80748	51
10	.54708	.83708	.56160	.82741	.57596	.81748	.59014	.80730	50
11	.54732	83692	.56184	.82724	.57619	.81731	.59037	.80713	49
12	.54756	.83676	.56208	.82708	.57643	.81714	.59061	.80696	48
13	.54781	.83660	.56232	.82692	.57667	.81698	.59084	.80679	47
14	.54805	.83645	.56256	.82675	.57691	.81681	.59108	.80662	46
15	.54829	.83629	.56280	.82659	.57715	.81664	.59131	.80644	45
16	.54854	.83613	.56305	.82643	.57738	.81647	.59154	.80627	44
17	.54878	.83597	.56329	.82626	.57762	.81631	.59178	.80610	43
18	.54902	.83581	.56353	.82610	.57786	.81614	.59201	.80593	42
19	.54927	.83565	.56377	.82593	.57810	.81597	.59225	.80576	41
20	.54951	.83549	.56401	.82577	.57833	.81580	.59248	.80558	40
21	.54975	.83533	.56425	.82561	.57857	.81563	.59272	.80541	39
22	.54999	.83517	.56449	.82544	.57881	.81546	.59295	.80524	38
23	.55024	.83501	.56473	.82528	.57904	.81530	.59318	.80507	37
24	.55048	.83485	.56497	.82511	.57928	.81513	.59342	.80489	36
25	.55072	.83469	.56521	.82495	.57952	.81496	.59365	.80472	35
26	.55097	.83453	.56545	.82478	.57976	.81479	.59389	.80455	34
27	.55121	.83437	.56569	.82462	.57999	.81462	.59412	.80438	33
28	.55145	.83421	.56593	.82446	.58023	.81445	.59436	.80420	32
29	.55169	.83405	.56617	.82429	.58047	.81428	.59459	.80403	31
30	.55194	.83389	.56641	.82413	.58070	.81412	.59482	.80386	30
31	.55218	.83373	.56665	.82396	.58094	.81395	.59506	.80368	29
32	.55242	.83356	.56689	.82340	.58118	.81378	.59529	.80351	28
33	.55266	.83340	.56713	.82363	.58141	.81361	.59552	.80334	27
34	.55291	.83324	.56736	.82347	.58165	.81344	.59576	.80316	26
35	.55315	.83308	.56760	.82330	.58189	.81327	.59599	.80299	25
36	.55339	.83292	.56784	.82314	.58212	.81310	.59622	.80282	24
37	.55363	.83276	.56808	.82297	.58236	.81293	.59646	.80264	23
38	.55388	.83260	.56832	.82281	.58260	.81276	.59669	.80247	22
39	.55412	.83244	.56856	.82264	.58283	.81259	.59693	.80230	21
40	.55436	.83228	.56880	.82248	.58307	.81242	.59716	.80212	20
41	.55460	.83212	.56904	.82231	.58330	.81225	.59739	.80195	19
42	.55484	.83195	.56928	.82214	.58354	.81208	.59763	.80178	18
43	.55509	.83179	.56952	.82198	.58378	.81191	.59786	.80160	17
44	.55533	.83163	.56976	.82181	.58401	.81174	.59809	.80143	16
45	.55557	.83147	.57000	.82165	.58425	.81157	.59832	.80125	15
46	.55581	.83131	.57024	.82148	.58449	.81140	.59856	.80108	14
47	.55605	.83115	.57047	.82132	.58472	.81123	.59879	.80091	13
48	.55630	.83098	.57071	.82115	.58496	.81106	.59902	.80073	12
49	.55654	.83082	.57095	.82098	.58519	.81089	.59926	.80056	11
50	.55678	.83066	.57119	.82082	.58543	.81072	.59949	.80038	10
51	.55702	.83050	.57143	.82065	.58567	.81055	.59972	.80021	9
52	.55726	.83034	.57167	.82048	.58590	.81038	.59995	.80003	8
53	.55750	.83017	.57191	.82032	.58614	.81021	.60019	.79986	7
54	.55775	.83001	.57215	.82015	.58637	.81004	.60042	.79968	6
55	.55799	.82985	.57238	.81999	.58661	.80987	.60065	.79951	5
56	.55823	.82969	.57262	.81982	.58684	.80970	.60089	.79934	4
57	.55847	.82953	.57286	.81965	.58708	.80953	.60112	.79916	3
58	.55871	.82936	.57310	.81949	.58731	.80936	.60135	.79899	2
59	.55895	.82920	.57334	.81932	.58755	.80919	.60158	.79881	1
60	.55919	.82904	.57358	.81915	.58779	.80902	.60182	.79864	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	56°		55°		54°		53°		

′	37° Sine	37° Cosine	38° Sine	38° Cosine	39° Sine	39° Cosine	40° Sine	40° Cosine	′
0	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	60
1	.60205	.79846	.61589	.78783	.62955	.77696	.64301	.76586	59
2	.60228	.79829	.61612	.78765	.62977	.77678	.64323	.76567	58
3	.60251	.79811	.61635	.78747	.63000	.77660	.64346	.76548	57
4	.60274	.79793	.61658	.78729	.63022	.77641	.64368	.76530	56
5	.60298	.79776	.61681	.78711	.63045	.77623	.64390	.76511	55
6	.60321	.79758	.61704	.78694	.63068	.77605	.64412	.76492	54
7	.60344	.79741	.61726	.78676	.63090	.77586	.64435	.76473	53
8	.60367	.79723	.61749	.78658	.63113	.77568	.64457	.76455	52
9	.60390	.79706	.61772	.78640	.63135	.77550	.64479	.76436	51
10	.60414	.79688	.61795	.78622	.63158	.77531	.64501	.76417	50
11	.60437	.79671	.61818	.78604	.63180	.77513	.64524	.76398	49
12	.60460	.79653	.61841	.78586	.63203	.77494	.64546	.76380	48
13	.60483	.79635	.61864	.78568	.63225	.77476	.64568	.76361	47
14	.60506	.79618	.61887	.78550	.63248	.77458	.64590	.76342	46
15	.60529	.79600	.61909	.78532	.63271	.77439	.64612	.76323	45
16	.60553	.79583	.61932	.78514	.63293	.77421	.64635	.76304	44
17	.60576	.79565	.61955	.78496	.63316	.77402	.64657	.76286	43
18	.60599	.79547	.61978	.78478	.63338	.77384	.64679	.76267	42
19	.60622	.79530	.62001	.78460	.63361	.77366	.64701	.76248	41
20	.60645	.79512	.62024	.78442	.63383	.77347	.64723	.76229	40
21	.60668	.79494	.62046	.78424	.63406	.77329	.64746	.76210	39
22	.60691	.79477	.62069	.78405	.63428	.77310	.64768	.76192	38
23	.60714	.79459	.62092	.78387	.63451	.77292	.64790	.76173	37
24	.60738	.79441	.62115	.78369	.63473	.77273	.64812	.76154	36
25	.60761	.79424	.62138	.78351	.63496	.77255	.64834	.76135	35
26	.60784	.79406	.62160	.78333	.63518	.77236	.64856	.76116	34
27	.60807	.79388	.62183	.78315	.63540	.77218	.64878	.76097	33
28	.60830	.79371	.62206	.78297	.63563	.77199	.64901	.76078	32
29	.60853	.79353	.62229	.78279	.63585	.77181	.64923	.76059	31
30	.60876	.79335	.62251	.78261	.63608	.77162	.64945	.76041	30
31	.60899	.79318	.62274	.78243	.63630	.77144	.64967	.76022	29
32	.60922	.79300	.62297	.78225	.63653	.77125	.64989	.76003	28
33	.60945	.79282	.62320	.78206	.63675	.77107	.65011	.75984	27
34	.60968	.79264	.62342	.78188	.63698	.77088	.65033	.75965	26
35	.60991	.79247	.62365	.78170	.63720	.77070	.65055	.75946	25
36	.61015	.79229	.62388	.78152	.63742	.77051	.65077	.75927	24
37	.61038	.79211	.62411	.78134	.63765	.77033	.65100	.75908	23
38	.61061	.79193	.62433	.78116	.63787	.77014	.65122	.75889	22
39	.61084	.79176	.62456	.78098	.63810	.76996	.65144	.75870	21
40	.61107	.79158	.62479	.78079	.63832	.76977	.65166	.75851	20
41	.61130	.79140	.62502	.78061	.63854	.76959	.65188	.75832	19
42	.61153	.79122	.62524	.78043	.63877	.76940	.65210	.75813	18
43	.61176	.79105	.62547	.78025	.63899	.76921	.65232	.75794	17
44	.61199	.79087	.62570	.78007	.63922	.76903	.65254	.75775	16
45	.61222	.79069	.62592	.77988	.63944	.76884	.65276	.75756	15
46	.61245	.79051	.62615	.77970	.63966	.76866	.65298	.75738	14
47	.61268	.79033	.62638	.77952	.63989	.76847	.65320	.75719	13
48	.61291	.79016	.62660	.77934	.64011	.76828	.65342	.75700	12
49	.61314	.78998	.62683	.77916	.64033	.76810	.65364	.75680	11
50	.61337	.78980	.62706	.77897	.64056	.76791	.65386	.75661	10
51	.61360	.78962	.62728	.77879	.64078	.76772	.65408	.75642	9
52	.61383	.78944	.62751	.77861	.64100	.76754	.65430	.75623	8
53	.61406	.78926	.62774	.77843	.64123	.76735	.65452	.75604	7
54	.61429	.78908	.62796	.77824	.64145	.76717	.65474	.75585	6
55	.61451	.78891	.62819	.77806	.64167	.76698	.65496	.75566	5
56	.61474	.78873	.62842	.77788	.64190	.76679	.65518	.75547	4
57	.61497	.78855	.62864	.77769	.64212	.76661	.65540	.75528	3
58	.61520	.78837	.62887	.77751	.64234	.76642	.65562	.75509	2
59	.61543	.78819	.62909	.77733	.64256	.76623	.65584	.75490	1
60	.61566	.78801	.62932	.77715	.64279	.76604	.65606	.75471	0
′	52° Cosine	52° Sine	51° Cosine	51° Sine	50° Cosine	50° Sine	49° Cosine	49° Sine	′

′	41° Sine	41° Cosine	42° Sine	42° Cosine	43° Sine	43° Cosine	44° Sine	44° Cosine	′
0	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.65694	.75395	.66999	.74227	.68285	.73056	.69549	.71853	56
5	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosine 48°	Sine 48°	Cosine 47°	Sine 47°	Cosine 46°	Sine 46°	Cosine 45°	Sine 45°	′

′	0° Tan.	0° Co-tan.	1° Tan.	1° Co-tan.	2° Tan.	2° Co-tan.	3° Tan.	3° Co-tan.	′
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	[illegible]	[illegible]	60
1	.00029	3437.750	.01775	56.3506	.03521	28.3994	[illegible]	[illegible]	59
2	.00058	1718.870	.01804	55.4415	.03550	28.1664	[illegible]	[illegible]	58
3	.00087	1145.920	.01833	54.5613	.03579	27.9372	[illegible]	[illegible]	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	[illegible]	[illegible]	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	[illegible]	[illegible]	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	[illegible]	[illegible]	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	[illegible]	[illegible]	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	[illegible]	[illegible]	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	[illegible]	[illegible]	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	[illegible]	[illegible]	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	[illegible]	[illegible]	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	[illegible]	[illegible]	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	[illegible]	[illegible]	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	[illegible]	[illegible]	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	[illegible]	[illegible]	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	[illegible]	[illegible]	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	[illegible]	[illegible]	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	[illegible]	[illegible]	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	[illegible]	[illegible]	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	[illegible]	[illegible]	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	[illegible]	[illegible]	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	[illegible]	[illegible]	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	[illegible]	[illegible]	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	[illegible]	[illegible]	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	[illegible]	[illegible]	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	[illegible]	[illegible]	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	[illegible]	[illegible]	33
28	.00814	122.774	.02560	39.0568	.04308	23.2137	[illegible]	[illegible]	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	[illegible]	[illegible]	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	[illegible]	[illegible]	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	[illegible]	[illegible]	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	[illegible]	[illegible]	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	[illegible]	[illegible]	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	[illegible]	[illegible]	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	[illegible]	[illegible]	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	[illegible]	[illegible]	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	[illegible]	[illegible]	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	[illegible]	[illegible]	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	[illegible]	[illegible]	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	[illegible]	[illegible]	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	[illegible]	[illegible]	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	[illegible]	[illegible]	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	[illegible]	[illegible]	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	[illegible]	[illegible]	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	[illegible]	[illegible]	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	[illegible]	[illegible]	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	[illegible]	[illegible]	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	[illegible]	[illegible]	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	[illegible]	[illegible]	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	[illegible]	[illegible]	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	[illegible]	[illegible]	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	[illegible]	[illegible]	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	[illegible]	[illegible]	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	[illegible]	[illegible]	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	[illegible]	[illegible]	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	[illegible]	[illegible]	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	[illegible]	[illegible]	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	[illegible]	[illegible]	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	[illegible]	[illegible]	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	[illegible]	[illegible]	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	89°		88°		87°		86°		

′	4° Tan.	4° Co-tan.	5° Tan.	5° Co-tan.	6° Tan.	6° Co-tan.	7° Tan.	7° Co-tan.	′
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33154	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Co-tan. 85°	Tan. 85°	Co-tan. 84°	Tan. 84°	Co-tan. 83°	Tan. 83°	Co-tan. 82°	Tan. 82°	′

	8°		9°		10°		11°		
′	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	′
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18383	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.83605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91235	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20025	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	81°		80°		79°		78°		

′	12°		13°		14°		15°		′
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27920	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03075	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	77°		76°		75°		74°		

′	16° Tan.	16° Co-tan.	17° Tan.	17° Co-tan.	18° Tan.	18° Co-tan.	19° Tan.	19° Co-tan.	′
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35117	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	[illegible]	[illegible]	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	[illegible]	[illegible]	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	[illegible]	[illegible]	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	[illegible]	[illegible]	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	[illegible]	[illegible]	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	[illegible]	[illegible]	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	[illegible]	[illegible]	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	[illegible]	[illegible]	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	[illegible]	[illegible]	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	[illegible]	[illegible]	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	[illegible]	[illegible]	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	[illegible]	[illegible]	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	[illegible]	[illegible]	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	[illegible]	[illegible]	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	[illegible]	[illegible]	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	[illegible]	[illegible]	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	[illegible]	[illegible]	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	[illegible]	[illegible]	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	[illegible]	[illegible]	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	[illegible]	[illegible]	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	73°		72°		71°		70°		

′	20° Tan.	20° Co-tan.	21° Tan.	21° Co-tan.	22° Tan.	22° Co-tan.	23° Tan.	23° Co-tan.	′
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37124	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38862	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	Co-tan. 69°	Tan. 69°	Co-tan. 68°	Tan. 68°	Co-tan. 67°	Tan. 67°	Co-tan. 66°	Tan. 66°	′

4°	25°		26°		27°		
Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	
2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
2.16090	.48414	2.06553	.50587	1.97680	.52798	1.89400	10
2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
2.15268	.48593	2.05790	.50769	1.96969	.52984	1.88734	5
2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
5°	64°		63°		62°		

	28°		29°		30°		31°		
	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1. 6209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65534	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56500	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58904	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76630	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	
	61°		60°		59°		58°		

	32°		33°		34°		35°		
′	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	′
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65023	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72166	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	57°		56°		55°		54°		

′	36° Tan.	36° Co-tan.	37° Tan.	37° Co-tan.	38° Tan.	38° Co-tan.	39° Tan.	39° Co-tan.	′
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36133	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Co-tan. 53°	Tan. 53°	Co-tan. 52°	Tan. 52°	Co-tan. 51°	Tan. 51°	Co-tan. 50°	Tan 50°	′

	40°		41°		42°		43°		
′	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	′
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85307	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	49°		48°		47°		46°		

′	44° Tan.	Co-tan.	′	′	44° Tan.	Co-tan.	′	′	44° Tan.	Co-tan.	′
0	.96569	1.03553	60	21	.97756	1.02295	39	41	.98901	1.01112	19
1	.96625	1.03493	59	22	.97813	1.02236	38	42	.98958	1.01053	18
2	.96681	1.03433	58	23	.97870	1.02176	37	43	.99016	1.00994	17
3	.96738	1.03372	57	24	.97927	1.02117	36	44	.99073	1.00935	16
4	.96794	1.03312	56	25	.97984	1.02057	35	45	.99131	1.00876	15
5	.96850	1.03252	55	26	.98041	1.01998	34	46	.99189	1.00818	14
6	.96907	1.03192	54	27	.98098	1.01939	33	47	.99247	1.00759	13
7	.96963	1.03132	53	28	.98155	1.01879	32	48	.99304	1.00701	12
8	.97020	1.03072	52	29	.98213	1.01820	31	49	.99362	1.00642	11
9	.97076	1.03012	51	30	.98270	1.01761	30	50	.99420	1.00583	10
10	.97133	1.02952	50	31	.98327	1.01702	29	51	.99478	1.00525	9
11	.97189	1.02892	49	32	.98384	1.01642	28	52	.99536	1.00467	8
12	.97246	1.02832	48	33	.98441	1.01583	27	53	.99594	1.00408	7
13	.97302	1.02772	47	34	.98499	1.01524	26	54	.99652	1.00350	6
14	.97359	1.02713	46	35	.98556	1.01465	25	55	.99710	1.00291	5
15	.97416	1.02653	45	36	.98613	1.01406	24	56	.99768	1.00233	4
16	.97472	1.02593	44	37	.98671	1.01347	23	57	.99826	1.00175	3
17	.97529	1.02533	43	38	.98728	1.01288	22	58	.99884	1.00116	2
18	.97586	1.02474	42	39	.98786	1.01229	21	59	.99942	1.00058	1
19	.97643	1.02414	41	40	.98843	1.01170	20	60	1	1	0
20	.97700	1.02355	40								
′	Co-tan. 45°	Tan.	′	′	Co-tan. 45°	Tan.	′	′	Co-tan. 45°	Tan.	′

6. Complete the following table using the table of sines cosines, etc.

1. Sin 30°
2. Cos 60°
3. Sin 20° 10′
4. Cos 69° 50′
5. Cos 36°
6. Sin 54°
7. Tan 12°
8. Cot 78°
9. Cot 48°
10. Tan 42°
11. Cos 27°
12. Sin 63°
13. Tan 45°
14. Cot 45°
15. Sin 20°
16. Cos 70°

From the above table what do you find about the sin 30° and the cos 60°, [that is, the cos of (90° − 30°)], also about the sin 20° 10′ and the cos of (90° − 20° 10′), that is, the cos 69° 50′; about the tan 12° and the cot of (90° − 12°) or cot 78°. In other words, this is true.

The sine of any angle less than 90° equals the co-function or cosine of (90° minus that angle), also, the tangent of any angle less than 90°, for example, the tangent of 12° equals the co-function or cotangent of (90° − 12°)

or equals the cot of 78°. In the same way the sine 20° 10′ = cos. 69° 50′, the cot 48° = tan (90° − 48°) or tan 42° the cos 30° = sin. (90° − 30°) or sin 60°, etc. This is a good rule to remember.

62. Line Values of Functions.—The values of the sine, cosine, tangent and cotangent of angles can be represented by the lengths of lines as follows: In Fig. 109 the circle with center O is drawn with a radius (one-half the diameter) equal to 1 in. CD is a horizontal and EF a vertical diameter. The radius OA makes any convenient angle. "a" with radius OC and from point A line AB is drawn at right angles to line OC making the angle at B equal to 90° and giving the right triangle OAB.

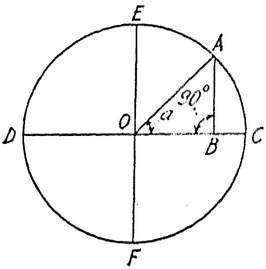


FIG. 109.

From the definition of the sine $\left(= \frac{\text{opposite side}}{\text{hypothenuse}}\right)$ we have in the right triangle OAB. The sine of angle $a = \frac{\text{side } AB}{\text{hyp. } OA}$. But $OA = 1$ by construction (since we made the circle with $OA = 1$ in.). Therefore sine $a = \frac{\text{side } AB}{1}$ or side AB. Hence the sine of a is equal to the length of the line AB. In the figure, AB measures .5 of an inch. Therefore sine a = .5. Notice that if angle a were *smaller* the line AB would be smaller, that is, sine a would be smaller. If angle a were *larger*, line AB would be larger and therefore sine a would be larger. When angle a = 0°, line AB = 0 in. and sine a = 0, that is, sine 0° = 0. When a = 90°, line AB = line OE in length or 1 in. and therefore sine 90° = 1. There

fore for angle between 0° and 90° the sine *increases* when the angle *increases.*

Referring again to Fig. 109, in the right triangle OAB the cosine of angle $a = \frac{\text{adjacent side}}{\text{hypothenuse}} = \frac{\text{side } OB}{\text{hyp. } OA} = \frac{\text{side } OB}{1}$ or side OB.

That is, the line OB equals to scale the *cosine* of angle a. When angle $a = 0$, $OB = OC$, and cos $a = 1$. When angle $a = 90°$, line $OB = 0$, and cosine $90° = 0$.

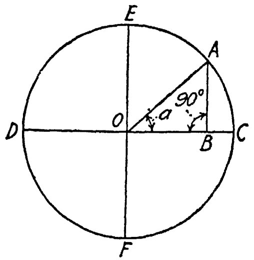


Fig. 110.

Therefore, for angles between 0° and 90° the *cosine decreases* as the *angle increases.* The figure on this sheet is like the preceding figure, page 157 drawn again here for convenience. Referring to right triangle OAB, the tangent of angle $a = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{side } AB}{\text{side } OB}$. Notice that as angle a *increases,* side AC (that is, the numerator of the fraction which is the tangent) *increases,* and at the same time side OB, the denominator of the fraction, *decreases.* Therefore as angle a *increases* the tangent *increases* and also if angle a *decreases* the tangent of angle a *decreases.*

In right triangle OAB the cotangent of angle $a = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{\text{side } OB}{\text{side } AB}$. Then as angle a *increases,* side OB *decreases* and side AB *increases;* therefore the fraction representing the cotangent *decreases.* As angle a decreases side OB increases and side AB *decreases,* and the value of the fraction *increases,* that is, the cotangent

increases. The following table for values of 0° and 90° angles should be carefully studied and understood from Fig. 110.

Angle	Sine	Cosine	Tangent	Cotangent
0°	0	1	0	Infinity
90°	1	0	Infinity	0

63. Values of the Sine, Cosine, Tangent, and Cotangent for the Common Angles.—In a 30°–60° right

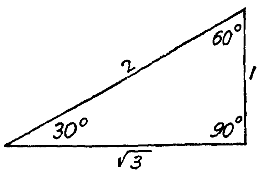


FIG. 111.

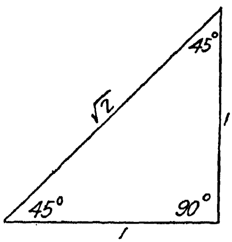


FIG. 112.

triangle shown in Fig. 111 the sides always have a ratio of 1, 2, and $\sqrt{3}$ as given. You can prove this for yourselves by carefully constructing such a triangle on the drawing board and measuring the lengths of the sides. Therefore from our definition of the sine, cosine, etc., of an angle we have

$$\text{Sin } 30° = \frac{\text{opp. side}}{\text{hyp.}} = \frac{1}{2} = .5$$

$$\text{Cos } 30° = \frac{\text{adj. side}}{\text{hyp.}} = \frac{\sqrt{3}}{2} = .866$$

$$\text{Tan } 30° = \frac{\text{opp. side}}{\text{adj. side}} = \frac{1}{\sqrt{3}} = .577$$

$$\text{Cot } 30° = \frac{\text{adj. side}}{\text{opp. side}} = \frac{\sqrt{3}}{1} = 1.732$$

$$\text{Sin } 60° = \frac{\text{opp. side}}{\text{hyp.}} = \frac{\sqrt{3}}{2} = .866$$

$$\text{Cos } 60° = \frac{\text{adj. side}}{\text{hyp.}} = \frac{1}{2} = .5$$

$$\text{Tan } 60° = \frac{\text{opp. side}}{\text{adj. side}} = \frac{\sqrt{3}}{1} = 1.732$$

$$\text{Cot } 60° = \frac{\text{adj. side}}{\text{opp. side}} = \frac{1}{\sqrt{3}} = .577$$

In a 45°–45° right triangle as shown in Fig. 112 the sides always have a ratio of 1, 1 and $\sqrt{2}$ as shown. Therefore for this figure:

$$\text{Sin } 45° = \frac{\text{opp. side}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = .707$$

$$\text{Cos } 45° = \frac{\text{adj. side}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = .707$$

$$\text{Tan } 45° = \frac{\text{opp. side}}{\text{adj. side}} = \frac{1}{1} = 1.00$$

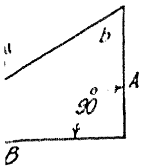
$$\text{Cot } 45° = \frac{\text{adj. side}}{\text{opp. side}} = \frac{1}{1} = 1.00$$

These values are tabulated below for reference.

Angle	Sine	Cosine	Tangent	Cotangent
30°	.5	.866	.577	1.732
45°	.707	.707	1.00	1.00
60°	.866	.5	1.732	.577

64. Methods of Working out Right Triangles.—In working out problems by the use of the sine, cosine, etc., we use one or the other, according to the sides of the triangle which are to be found and the sides which are

or example in Fig. 113 the hyp. = 10 in. and acute angle = 30°. We want to find side nd angle b. In any right triangle the sum of ute angles equals 90°, therefore angle b + and angle b = 90° − 30° or 60°. To find ave sine 30° = $\frac{\text{opp. side}}{\text{hyp.}} = \frac{\text{side } A}{10 \text{ in.}}$. If sine $\frac{A}{}$, side A = 10 × sine 30° (sine 30° = .5),



. 113.

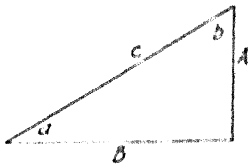


FIG. 114

le A = 10 × .5 or 5 in. To find side B osine as follows:

$$\cos 30° = \frac{\text{adj. side}}{\text{hyp.}} = \frac{\text{side } B}{10}$$

le B = 10 × cos 30°, (cos 30° = .866), B = 10 × .866 or 8.66 in.

explanation just given and using Fig. 114 following table. The first set of values orked out as a guide.

e a	Angle b	Side C	Side A	Side B
[illegible]	60°	16	8	13.9
[illegible]		24		
	30°	38		
[illegible]		7.5		
[illegible]		62		
	75°	35		

Draw a separate triangle for each set of values and put the lengths of sides and sizes of angles on each figure as well as putting them in the table.

If as in the triangle shown (Fig. 115) we had given angle $a = 30°$ and side $B = 8$, to find side A we use the tanget as follows: $\tan 30° = \frac{\text{side } A}{\text{side } B}$, therefore side A = side $B \times \tan 30°$ or side $A = 8 \times .577 = 4.62$.

If as in the triangle shown in Fig. 116 we have the side

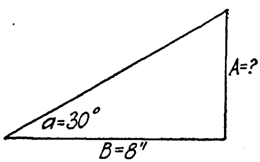


FIG. 115.

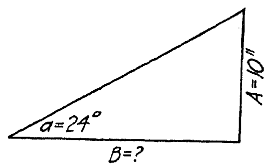


FIG. 116.

$A = 10$ and angle $a = 24°$ and wish to find side B we use the cotangent as follows:

$$\cot 24° = \frac{\text{side } B}{10}$$

Therefore side $B = 10 \times \cot 24°$ or side $B = 10 \times 2.25$ or 22.5. From the explanation just given and using Fig. 117 complete the following table.

	Angle a	Angle b	Side A	Side B
1		29°	16	
2	80°			8.4
3		36°	35	
4	15°			45
5		75°	7.5	
6	60°			24

When the base B and angle a are given (Fig. 118) to find the hypothenuse proceed as follows:

$$\cos 40^\circ = \frac{\text{side } B}{\text{hyp.}}$$

or

$$\cos 40^\circ = \frac{20}{\text{hyp.}}$$

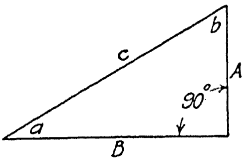


FIG. 117.

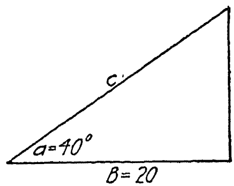


FIG. 118.

Therefore $\cos 40^\circ \times$ hyp. $= 20$ and from this the hyp. $=$

$$\frac{20}{\cos\ 40^\circ} = \frac{20}{.766} = 26.1$$

When the side A and angle a are given (Fig. 119) to find the hypothenuse proceed as follows:

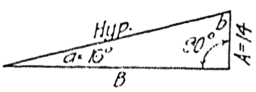


FIG. 119.

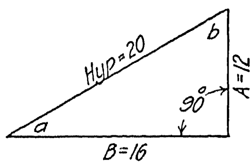


FIG. 120.

$$\sin\ 16^\circ = \frac{\text{side } A}{\text{hyp.}} \text{ or } \sin\ 16^\circ = \frac{14}{\text{hyp.}}$$

Therefore hyp. $\times \sin\ 16^\circ = 14$ and hyp. $= \dfrac{14}{\sin\ 16^\circ}$ or

$$\text{hyp.} = \frac{14}{.276} = 50.7$$

7. From the explanation just given fill in the following ta

	Angle a	Angle b	Side A	Side B
1	12°		10	
2		16° 30′		20
3	43°		15	
4		27° 10′		63
5	36°		23.8	
6		32°		42

When we have two sides of a right triangle and to find the angles we proceed as follows: In the tr shown in Fig. 120 $A = 12$, $B = 16$, hyp. $= 20$ find angles a and b.

Sin $a = \dfrac{\text{opp. side}}{\text{hyp.}} = \dfrac{12}{20} = \dfrac{3}{5} = .6$. If sin $a = .6$ angle a is one whose sine is .6. We therefore look tables for an angle whose sine is .6 and on page 14 the number .59995 which is the nearest one to .(the angle is therefore 36° 52′.

We can find angle a also from the rule

$$\cos a = \frac{\text{adj. side}}{\text{hyp.}} = \frac{16}{20} = \frac{8}{10} = .8$$

Looking in the tables for the angle whose cosine is . we find on page 142 the angle 36° 52′.

We can find angle a also from the tangent r follows:

$$\text{Tan } a = \frac{\text{opp. side}}{\text{adj. side}} = \frac{12}{16} = \frac{3}{4} = .75$$

and therefore (page 154) angle a = 36° 52′, or fro cotangent rule

$$\cot a = \frac{16}{12} = \frac{4}{3} = 1.33$$

and from the tables (page 154) angle a = 36° 52′.

Having obtained angle a from any one of the rules just given, angle b equals $90° -$ angle a, since angle $a +$ angle $b = 90°$.

8. From the explanation just given and using Fig. 121, find the value of the sine of angle a and the value of angle a, completing the following table.

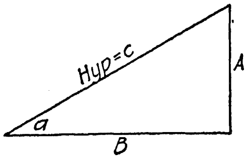


FIG. 121.

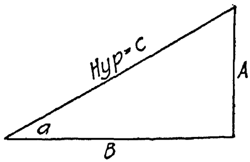


FIG. 122.

	Side A	Hyp.	Sine a	Angle a
1	25	50		
2	7	35		
3	28	38		
4	15	20		
5	12	32		

9. Using Fig. 122, find the value of the cosine of angle a and angle a, completing the following table.

	Side B	Hyp.	Cosine a	Angle a
1	10	20		
2	3	4		
3	30	75		
4	16	28		
5	14	32		

10. For Fig. 123, find the tangent of angle a and the value of angle a, completing the following table.

	Side A	Side B	Tangent a	Angle a
1	10	24		
2	36	70		
3	3	8		
4	56	40		
5	38	112		

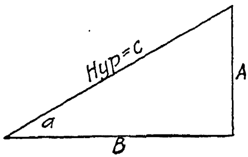


FIG. 123.

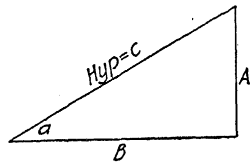


FIG. 124.

11. For Fig. 124, find the cotangent of angle a and the value of angle a, completing the following table.

	Side A	Side B	Cotangent a	Angle a
1	7	32		
2	27	38		
3	14	18		
4	12	16		
5	23	42		

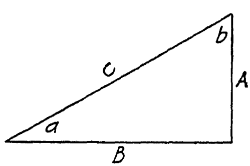


FIG. 125.

12. Using Fig. 125, complete the following table, drawing a separate triangle for each problem.

	Angle a	Angle b	Side A	Side B	Side C
1	60°				75
2		40°		35	
3		32°	15		
4	75°		20		
5	15°			30	
6			25		50

65. Applications of Triangular Functions.—Following are applications of the preceding rules to the solution of problems in daily shop work.

13. To find the depth of a V thread. Fig. 126 shows a section of the thread. To find the depth of the thread or distance CD

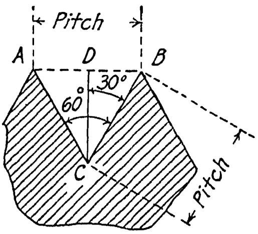


FIG. 126.

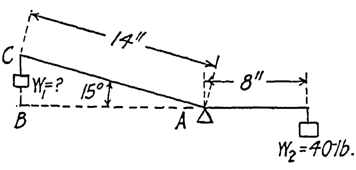


FIG. 127.

in triangle DBC. Side CD = side $CB \times \cos 30°$ = pitch $\times$ .866. The pitch in in. always equals $\frac{1}{\text{threads per in.}}$

Find the depth of V threads as follows: 8 per in., 20 per in., 12 per in., and 40 per in.

14. In a right triangle the hypothenuse is 20 in. and the height is to be twice the base. Find the height, base, and all the angles of the triangle. (NOTE.—Let the base be x in. and the height $2x$ in. The tangent of the base angle will then be $\frac{2x}{x}$ or 2.)

15. Fig. 127 shows a first-class lever. If W_2 = 40 lb. to find

W_1 we have the rule $40 \times 8 = W_1 \times$ distance AB, but length $AB = 14 \times \cos 15° = 14 \times .966 = 13.5$, therefore $40 \times 8 = W_1 \times 13.5$ and $W_1 = \frac{40 \times 8}{13.5} = 23.7$ lb. *Ans.*

16. With a first-class lever as in Fig. 128, find W_1 necessary to balance W_2 with lever arms as shown.

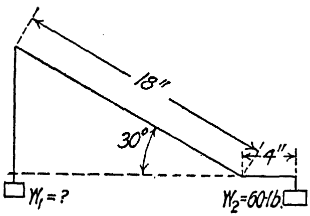


FIG. 128.

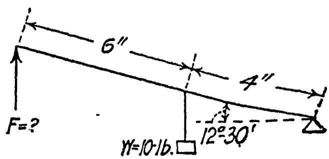


FIG. 129.

17. As per Fig. 129, find the force necessary at F to balance the weight of 10 lb. shown.

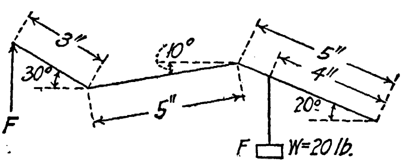


FIG. 130.

18. As shown in Fig. 130, find the force necessary at F to balance the 20 lb. What class of lever is this one?

19. What weight in lb. at W (Fig. 131) is required to set the safety valve so as to lift at 200 lb. per sq. in. pressure with a 2-in. diam. valve?

20. As per Fig. 132, what size valve is required with steam at 150 lb. per sq. in. to lift the weight of 80 lb. at W, with the lever arms as shown?

21. In a laying out job the distance CB Fig. 133 is taken as 16 in., the angle $a = 24°$. Find the distance between centers A and B.

22. The tangent of an angle is .45573. What are its sine, cotangent, and cosine?

23. For the wedge shown (Fig. 134) find the angle between its sides.

24. Find the angle between the sides of a wedge 1 ft. long, $\frac{7}{8}$ in. wide at one end and $\frac{1}{4}$ in. wide at the other end.

25. The sides of a wrought-iron wedge form an angle of 2° 23′. Find the taper (decrease in thickness) per ft. of length.

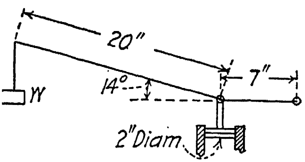


FIG. 131.

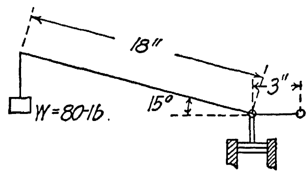


FIG. 132.

26. A reamer $1\frac{5}{8}$ in. at the small end is 16 in. long and has a aper of $\frac{3}{4}$ in. in. 12 in. ($\frac{3}{4}$ in. increase in diam. per ft.). What is he diameter at the large end and what is the angle between the wo sides?

27. Find the angle which the sides of a wedge make with one nother for a taper of $\frac{7}{8}$ in. per ft.

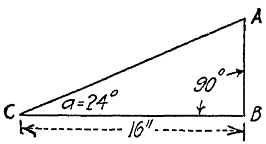


FIG. 133.

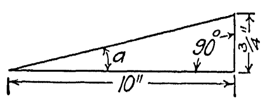


FIG. 134.

28. Fig. 135 shows a telegraph pole with a guy wire attached) ft. up on pole and anchored 10 ft. from base of pole. Find ngth of wire required. Allow 2 ft. extra for fastening wire. NOTE.—Find first angle *a*, then get side *C* from the sine or cosine le.)

29. Find the angle of incline for a grade of 3 per cent., and the mber of feet rise in a distance of 2 miles. (A 1 per cent. grade one which rises 1 ft. vertically for every 100 ft. horizontally.)

How many foot-pounds of work are required to lift a 600-ton train up this distance against gravity?

30. Find the average angle of incline for the roadbed from Altoona, Pa., to Gallitzin, Pa., the rise being about 980 ft. in 10 miles.

31. Two adjacent spokes in a wheel make with one another an angle whose cosine is .91355. What is the angle? How many spokes are there in the wheel?

32. A belt passes over two pulleys whose centers are 11 ft. apart. The pulley diameters are 18 in. and 42 in. Find the angle between the two sides of the belt.

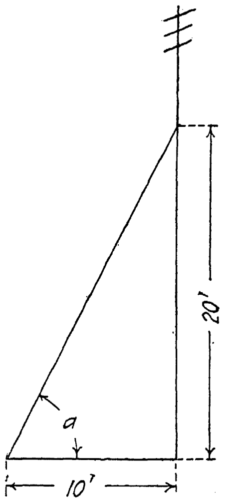


FIG. 135.

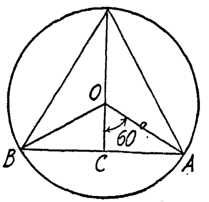


FIG. 136.

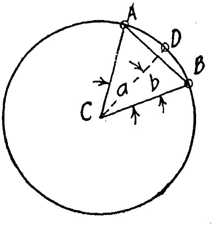


FIG. 137.

33. Find the angle between the sides of a belt connecting two pulleys 24 in. and 12 in. in diam. and with centers 6 ft. apart.

34. Find the length of a side of an equilateral triangle inscribed in a 24-in. diam. circle. Fig. 136 shows the circle and triangle, Drawing the radii OA and OB we have the triangle OAB. Line OC divides this triangle into two *equal right* triangles OAC and OCB.

ıngle $AOC = 60°$. To find side AB we find AC first, then $AB =$ $\times AC$. Now in triangle AOC

$$\frac{AC}{OA} = \text{sine } 60°$$

͞herefore $AC = OA \times \sin 60° = OA \times .866$

ʒut OA is the radius of the circle or 12 in.
͞herefore $AC = 12 \times .866 = 10.4$ in. and $AB = 2 \times AC = 2 \times$ 0.4 $= 20.8$ in. *Ans.*

35. Find the length of one side of each of the following equal-.ded inscribed figures.

Number of equal sides	Diam of circle in which figure is inscribed
5	18 in.
7	24 in.

36. As shown in Fig. 137, find the distance AB to be laid off for rilling two of eight equally spaced holes on an 8-in. diam. circle.

$$\text{angle } a = \frac{360°}{8} = 45°$$

$$\text{angle } b = \frac{45}{2} = 22\tfrac{1}{2}°.$$

'herefore in right triangle BCD

$$\frac{\text{side } BD}{\text{side } BC} = \sin \text{angle } b, \text{ therefore}$$

$$\begin{aligned}\text{side } BD &= \text{side } BC \times \sin b.\\ &= 4 \text{ in.} \times \sin 22\tfrac{1}{2}°.\\ &= 4 \times .383 = 1.532 \text{ in.}\\ \text{and } AB &= 2 \times BD = 2 \times 1.53 = 3.06 \text{ in.} \quad \textit{Ans.}\end{aligned}$$

37. Find the distance at which to set dividers for laying off 12 ʇually spaced holes on a 42-in. diam. circle.

38. What is the distance in in. between any two consecutive ›nters when 17 are spaced an equal distance apart on a 1-ft. .am. circle?

39. If the straight distance between two consecutive centers ı a 12-in. diam. circle is $1\tfrac{1}{2}$ in., what is the angle formed at the ›nter of the circle by the radii drawn to these two centers?

40. In the sketch, Fig. 138, showing a cross-sectional view of a

boiler shell 72-in. diam. with water line 20 in. above the center of shell, find the length of the water line "x" in in.

41. What is the taper per ft. of length for a wedge, the sides of which form an angle whose tangent is .0626?

42. In Fig. 139, diameter AB = 2 in., chords AC and AD are

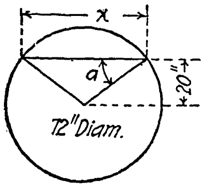


FIG. 138.

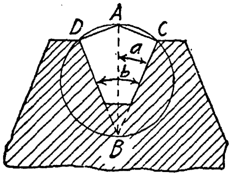


FIG. 139.

each ½ in. and angles C and D are each 90°. Find angle a and angle b. Angle b is that used in constructing the "acme" thread. Remember how this figure is drawn, to get this angle for an "acme thread."

43. In Fig. 140, find the length of the center lines of rafters A

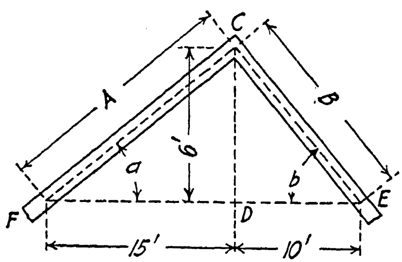


FIG. 140.

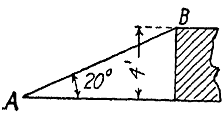


FIG. 141.

and B. (NOTE.—The tangent of angle $a = \frac{6}{15} = \frac{2}{5} = .4$. Find angle a in the tables. Then knowing a and the 15-ft. side of the triangle CDF find side A. In the same way to find side B, find angle b first, then work out the right triangle CDE for side B knowing angle b and the 10-ft. side.

44. A line shaft is 18 ft. above and 15 ft. to the left of a motor

shaft. Find the distance between these shafts by drawing the right triangle, using the tables to get one of the acute angles and then the sine or cosine rule to get the hypothenuse of the triangle or the answer.

45. Find the angle between the belt on two sides of pulleys 18 in. and 42 in. in diam. and centers 11 ft. apart. Make a sketch of the belt and pulleys.

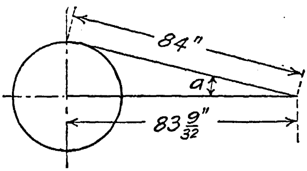


FIG. 142.

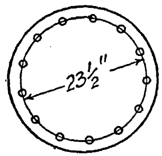


FIG. 143.

46. Find the length of plank required to reach from A to B, as per Fig. 141.

47. In Fig. 142 with crank pin on the top quarter the distance from center of driver to center of cross-head is $83\frac{9}{32}$ in. The main rod is 84 in. long. Find the angle a and the stroke of the engine.

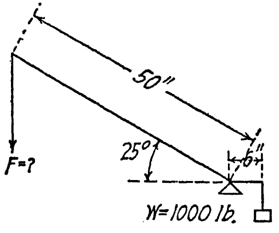


FIG. 144.

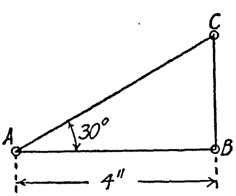


FIG. 145.

48. As in Fig. 143 find to what distance you would set dividers for laying off 26 equally spaced stud holes on a $23\frac{1}{2}$-in. diam. circle on a cylinder head.

49. As shown in Fig. 144, what force at F is necessary to lift the weight of 1000 lb.?

50. Three holes are to be laid out as in Fig. 145. Find distances BC and AC.

51. In Fig. 146, $\cos a = .955$. Find angle a and sides x and y.

52. The sides of a wrought-iron wedge form an angle of 2° 23′. What is the taper per ft. of length?

53. Find the taper per ft. of length of a wedge the sides of which form an angle whose tangent is .0625.

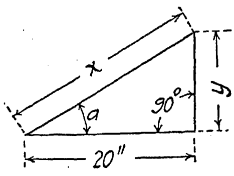


FIG. 146.

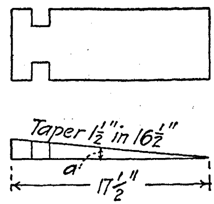


FIG. 147.

54. The shank on a drill socket has a taper of ⅝ in. in 12 in., is ⅝ in. in diam. at the small end and 5 in. long. What is its diameter at the large end (to the nearest 64th of an in.) and the angle between the sides?

55. If a piece of work is to be planed to a taper of $\frac{1}{16}$ in. in 1 in. for a length of 22½ in., how high would you block up the thin end?

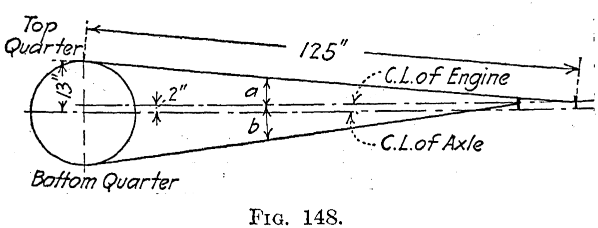


FIG. 148.

56. To what distance would you set dividers for laying off 4 equally spaced centers on an 8-in. diam. circle?

57. In Fig. 147 showing two views of a wedge find angle a in degrees and minutes. How high would you block up the thin end for planing the tapered surface?

58. By what distance in in. is the center of the cross-head (Fig. 148) drawn back of the middle of its stroke when the crank pin is on both the top and bottom quarters?

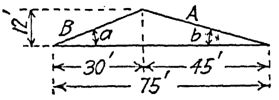


FIG. 149.

59. In Fig. 149, find the value of angles a and b and sides A and B.

60. For Fig. 150, find the distance between the centers A and B.

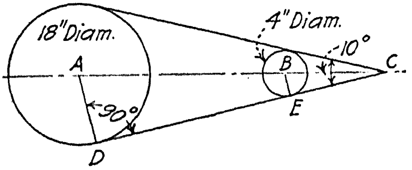


FIG. 150.

(NOTE.—Find first the hypothenuse of each of the right triangles ADC and BEC.)

61. Fig. 151 shows a cross-sectional view of a boiler shell 76¾

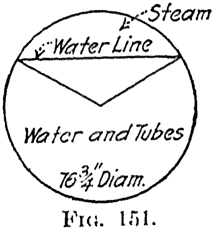


FIG. 151.

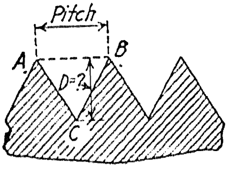


FIG. 152.

in. in diam. and the water line is 21½ in. above the center line of the boiler. Find (1) the area of the steam segment, (2) the area of the water segment, and (3) the *net* water area when there are 465 2-in. outside diam. (*O. D.*) tubes.

62. Find the depth of a *V* thread in terms of the number of threads per in. Triangle *ABC*, Fig. 152, is equiangular and therefore equilateral.

63. Fig. 153 represents a cross-sectional view of a boiler shell 7 ft. 6 in. in diam. and has 482 1⅝-in. *O. D.* tubes, the ratio of the height of the steam segment to the height of the water

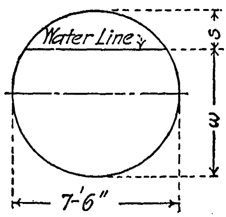


FIG. 153.

segment or S to W is as 2 is to 7, thus $\frac{S}{W} = \frac{2}{7}$. Find (1) the height and area of the steam segment; (2) the height and area of the water segment; (3) the net area of the water segment after deducting area of the tubes.

64. Determine the distance in in. between the center of the

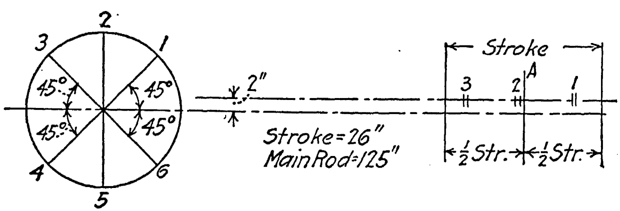


FIG. 154.

cross-head and the center of the stroke or point "*A*," Fig. 154, when the crank pin occupies the six different positions marked 1–2–3–4–5 and 6. The center line of the cylinder is 2 in. above the center line of the axle. How would these distances be changed, first by shortening the main rod, and second by lengthening the main rod?

CHAPTER XVIII

THE MEASUREMENT OF OBLIQUE TRIANGLES

66. Methods of Working out Oblique Triangles.—In the following problems we are to work out triangles which have *no right angles*, that is *oblique triangles*. How we work out these oblique triangles depends on which sides and angles and how many of either are given, and which sides and angles we wish to find.

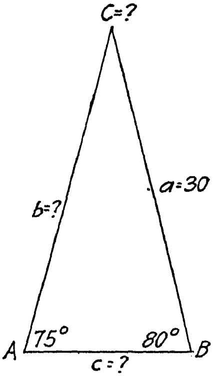


FIG. 155.

In any problem we will have one of the four following cases.

1. When one side and two angles are known, to find the remaining two sides and angle.

2. When two sides and their included angle are known, to find the other side and other two angles.

3. When only the three sides are known, to find all three angles.

4. When two sides and the angle opposite only one of these sides is known, to find the other side and other two angles.

CASE I

Example.—Given, any one side and any two angles of the triangle. To find the other two sides and angle.

In Fig. 155 suppose we have given angle $A = 75°$,

angle $B = 80°$ and side $a = 30$. To find angle C and sides b and c we proceed as follows:

In every triangle the three angles added together equal 180°. Therefore angle A + angle B + angle C = 180°. But angle $A = 75°$ and angle $B = 80°$. Therefore $75° + 80°$ + angle $C = 180°$ and angle $C = 180° - 75° - 80°$ or 25°.

To find the sides b and c we have the rule. In any triangle any two sides have the same ratio as the *sines* of their opposite angles. Therefore the figure shows

$$\frac{\text{side } b}{\text{side } a} = \frac{\text{sine angle } B}{\text{sine angle } A}$$

also

$$\frac{\text{side } c}{\text{side } a} = \frac{\text{sine angle } C}{\text{sine angle } A}$$

using the last rule we have

$$\frac{\text{side } c}{30} = \frac{\text{sine } 25°}{\text{sine } 75°} \text{ or } \frac{\text{side } c}{30} = \frac{.423}{.966}$$

$$\text{from which side } c = \frac{30 \times .423}{.966} \text{ or } 13.14 \text{ in.}$$

also from the rule that

$$\frac{\text{side } b}{\text{side } a} = \frac{\text{sine angle } B}{\text{sine angle } A}$$

$$\frac{\text{side } b}{30} = \frac{\text{sine } 80°}{\text{sine } 75°}$$

From which

$$\frac{\text{side } b}{30} = \frac{.985}{.966}$$

$$\text{and side } b = \frac{30 \times .985}{.966} \text{ or } 30.6 \text{ in.}$$

Example.—In Fig. 156 angle $A = 56°\ 10'$ angle $B = 64°\ 30'$ and side $b = 21$. We want to find angle C and sides a and c.

$$\text{Angle } C = 180° - (\text{angle } A + \text{angle } B)$$
$$= 180° - (56° \ 10' + 64° \ 30')$$
$$= 59° \ 20'$$

$$\frac{\text{side } a}{\text{side } b} = \frac{\text{sine } A}{\text{sine } B}$$

$$\frac{\text{side } a}{21} = \frac{.830}{.903}$$

Therefore $\quad \text{side } a = \dfrac{21 \times .830}{.903}$ or 19.3

To find side c, we have the rule:

$$\frac{\text{side } c}{\text{side } b} = \frac{\sin C}{\sin B} \text{ therefore } \frac{\text{side } c}{21} = \frac{.860}{.903}$$

from which, $\quad \text{side } c = \dfrac{21 \times .860}{.903}$ or 20.0

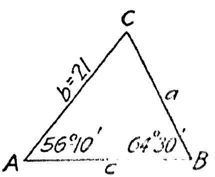


FIG. 156.

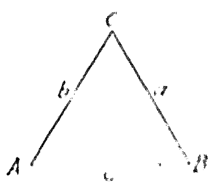


FIG. 157.

1. From the explanation just given and using Fig. 157, complete the following table, drawing a separate triangle for each problem. The answers are given to the first two problems to help in understanding them.

	Angle A	Angle B	Angle C	Side a	Side b	Side c
1	60°	65° 20′	[54° 40′]	18 m	[18.9 m]	[16.9 m]
2	80°	30° 30′	[69° 30′]	[46.6 m]	24 m	[illegible]
3	70°		40° 50′			[illegible]
4		48°	72°		[illegible]	
5	37°	87° 15′		[illegible]		
6		12° 16′	85°			[illegible]
7		30°	60°	[illegible]		

CASE II

Example.—Given, any two sides and their included angle. To find the remaining side and remaining angles.

In Fig. 158 we have given side $a = 86$, side $c = 72$, and angle $B = 74° 20'$. In order to find side b and angles A and C we use the following rules.

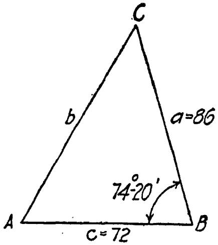


FIG. 158.

$$\frac{\tan \tfrac{1}{2}(\text{angle } A - \text{angle } C)}{\tan \tfrac{1}{2}(\text{angle } A + \text{angle } C)} = \frac{\text{side } a - \text{side } c}{\text{side } a + \text{side } c}$$

Now angle A + angle B + angle $C = 180°$

Therefore Angle A + Angle $C = 180°$ − Angle B or $180° - 74° 20'$ or $105° 40'$.

Then using the rule above, we have

$$\frac{\tan \tfrac{1}{2}(\text{angle } A - \text{angle } C)}{\tan \tfrac{1}{2}(105° 40')} = \frac{86 - 72}{86 + 72}$$

or $\tan \tfrac{1}{2}(\text{angle } A - \text{angle } C) = \tan \tfrac{1}{2}(105° 40') \times \frac{14}{158}$.

The $\tan \tfrac{1}{2}(105° 40') \times \frac{14}{158} = \tan 52° 50' \times \frac{14}{158}$ or $\frac{1.319 \times 14}{158}$. This fraction worked out equals .117.

If $\tan \tfrac{1}{2}(\text{angle } A - \text{angle } C) = .117$ we find from the table of tangents that $\tfrac{1}{2}(\text{angle } A - \text{angle } C) = 6° 40'$ and angle A − angle $C = 2 \times [6° 40']$ or $13° 20'$, but angle A + angle $C = 105° 40'$. Adding, $2 \times$ angle $A = 119°$ and angle $A = 59° 30'$.

Angle C = 180° − angle A − angle B or
180° − 59° 30′ − 74° 20′ or 46° 10′
To find side b we have as in Case I.

$$\frac{\text{side } b}{\text{side } a} = \frac{\sin B}{\sin A}$$

That is

$$\frac{\text{side } b}{86} = \frac{\sin 74° 20'}{\sin 59° 30'}$$

Therefore
$$\text{side } b = \frac{86 \times .963}{.862} = 96.1$$

2. From the explanation just given and using Fig. 159, complete the following table, drawing a separate triangle for each problem.

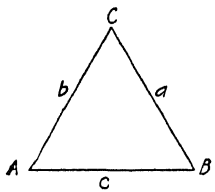


FIG. 159.

The answers in parenthesis( ) are given for the first two problem to help in understanding them.

	Angle A	Angle B	Angle C	Side a	Side b	Side c
1	(116° 10′)	40° 20′	(23° 30′)	72	(51.9)	32
2	58° 40′	(70°)	(51° 20′)	(5.90)	65	5.4
3			70°	120	110	
4	36° 40′				98	103
5		86°		18		24
6			98° 20′	7.92	12.3	
7	88° 40′				2.35	4.25
8		23°		56		63

CASE III

Example.—Given, the three sides of a triangle. To find the three angles. In Fig. 160 we have given the sides as follows: side $a = 4$, side $b = 6$, side $c = 8$. To find angles A, B and C we use the following rules in which $s = \dfrac{\text{side } a + \text{side } b + \text{side } c}{2}$ that is *one-half* the sum of the three sides of the triangle.

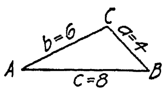


FIG. 160

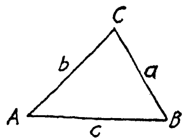


FIG. 161.

$$(1)\ \cos \tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}} \qquad (2)\ \cos \tfrac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}}$$

$$(3)\ \cos \tfrac{1}{2}C = \sqrt{\frac{s(s-c)}{ab}}$$

In Fig. 160 we have $s = \dfrac{\text{side } a + \text{side } b + \text{side } c}{2} = \dfrac{4+6+8}{2}$ or 9, and using rule (1) above:

$\cos \tfrac{1}{2}A = \sqrt{\dfrac{9(9-4)}{6 \times 8}}$ which equals $\sqrt{\dfrac{9 \times 5}{48}}$ or .9683. If $\cos \tfrac{1}{2}A = .9683$ from the table of cosines we find $\tfrac{1}{2}A = 14° \ 28'$, and therefore $A = 28° \ 56'$.

By rule (2) above, $\cos \tfrac{1}{2}B = \sqrt{\dfrac{9(9-6)}{4 \times 8}}$ which equals $\sqrt{\dfrac{27}{32}}$ or .9185, and from the tables $\tfrac{1}{2}B = 23° \ 17'$. Therefore $B = 46° \ 34'$. Then angle $C = 180° -$ (angle $A +$

angle B) or $180° - (28° 56' + 46° 34')$ which equals $104° 30'$.

3. Using Fig. 161 and from the explanation just given complete the following table, drawing a separate triangle for each problem. The answers to the first problem are already given.

	Side a	Side b	Side c	Angle A	Angle B	Angle C
1	3	5	6	(29° 56′)	(56° 16′)	(93° 48′)
2	4	6	5			
3	12	10	9			
4	5.6	4.8	3.7			
5	5.8	6.5	8.4			
6	60	80	48			
7	70	62	39			
8	56	43	36			

Put on each triangle the lengths of the sides given and the values of the angles found.

CASE IV

Example.—Given any two sides of the triangle and the angle opposite one of them To find the remaining side and other two angles. Suppose, as in Fig. 162, we have given side $a = 24$, side $b = 30.5$ and angle $A = 31° 20'$. To find angle B we have by Case I:

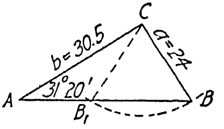


FIG. 162.

$$\frac{\sin B}{\sin A} = \frac{\text{side } b}{\text{side } a} \text{ or } \sin B = \frac{30.5 \times .520}{24} \text{ or } .6608$$

and from the table of sines we find angle $B = 41° 22'$. But an angle and its *supplement* have the same sine, the supplement of an angle being that angle which added to the given angle will equal 180 degrees. For example,

suppose we have given an angle of $38° 40'$, then $180° - 38° 40' = 141° 20'$ and $141° 20'$ is the supplement of the angle $38° 40'$.

It is possible, therefore, to have two values for angle B in this problem. Hence another value for angle B in this problem is $180° - 41° 22'$ or $138° 38'$. Call this value B_1; and since we have two values for angle B, we must have two triangles, each having an angle $A = 31° 20'$, a side $a = 24$, and a side $b = 30.5$, and hence two values for angle C and side c. Call these values C and C_1. Then $C = 180° - (A + B)$ or $180° - (31° 20' + 41° 22')$ which equals $107° 18'$; also $C_1 = 180° - (A + B_1)$ or $180° - (31° 20' + 138° 38')$ or $10° 2'$.

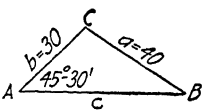


FIG. 163.

We have now to find side c and side c_1. By Case I we have

$$\frac{\text{side } c}{\text{side } a} = \frac{\text{sine } C}{\text{sine } A} \text{ and } \frac{\text{side } c_1}{\text{side } a} = \frac{\text{sine } C_1}{\text{sine } A}$$

Therefore $\quad \text{side } c = \dfrac{24 \times .95476}{.520}$ or 44.06

and $\quad \text{side } c_1 = \dfrac{24 \times .17422}{.520}$ or 8.04

We will not always have two solutions, that is, two values for the unknown angles and sides. For instance, suppose as in Fig. 163 we had given side $a = 40$, side $b = 30$ and angle $A = 45° 30'$. Then by Case I we have

$$\frac{\text{sine } B}{\text{sine } A} = \frac{\text{side } b}{\text{side } a} \text{ or sine } B = \frac{30 \times .71325}{40} \text{ or } .5349$$

and from the table of sines we have $B = 32° 20'$.

The supplement of angle B is $180° - 32° 20'$ or $147° 40'$, that is, angle B_1. But the angles $A + B_1 = 45° 30'$

$+ 147° 40'$ or $193° 10'$ which is impossible, since the sum of any two angles of a triangle must be less than $180°$, hence we have only one solution to the problem: Angle $C = 180° - (A + B)$ or $180° - (45° 30' + 32° 20')$ or $102° 10'$, since

$$\frac{\text{side } c}{\text{side } a} = \frac{\text{sine } C}{\text{sine } A}, \text{ side } c = \frac{40 \times .97754}{.71325} \text{ or } 54.8$$

Again, suppose as shown in Fig. 164 we had given side $a = 32$, side $b = 44$, and angle $B = 112° 30'$. Then by Case I we have $\frac{\text{sine } A}{\text{sine } B} = \frac{\text{side } a}{\text{side } b}$.

Now sine $112° 30'$ = sine $(180° - 112° 30')$ or sine $67° 30'$ (since the sine of any angle less than $180°$ equals the sine of $180°$ minus that angle). Therefore sine $A = \frac{32 \times .92388}{44}$ or .67191, and from the table $A = 42° 13'$. Angle $C = 180° - (A + B)$ or $180° - (42° 13' + 112° 30')$ or $25° 17'$; also since $\frac{\text{side } c}{\text{side } a} = \frac{\text{sine } C}{\text{sine } A}$, then

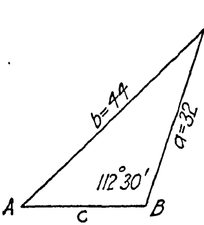


FIG. 164.

$$\text{side } c = \frac{32 \times .42709}{.67191} \text{ or } 20.34.$$

It is evident, since the given angle is greater than $90°$, that there is only one solution to this problem.

We now have the following rules by which we may tell at a glance how many solutions there are to a problem under this case (Case IV).

1. If the side opposite the given angle is less than the other given side there are two solutions.

2. If the side opposite the given angle is greater than the other given side there is only one solution.

3. If the given angle is greater than 90° there is only one solution.

4. From the explanation just given and using Fig. 165 as a guide complete the following table, drawing a separate triangle for each problem. The answers to the first problem are already worked out.

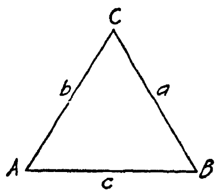


FIG. 165.

	Angle A	Angle B	Angle C	Side a	Side b	Side c
1	32° 10′	$B = 45°13'$ $B_1 = 134°47'$	$C = 102°37'$ $C_1 = 13°3'$	6	8	$c = 11$ $c_1 = 2.54$
2			26° 40′		10	7
3		102° 50′			73	65
4			39° 10′	28		33
5		59°		16	14	
6		120° 50′			48	42
7			83° 20′	47		51
8	128° 30′			38		30

67. Applications of Triangular Functions to Oblique Triangles.—Following are applications to the solution of practical problems of the preceding rules for oblique triangles.

5. In Fig. 166 is shown the center lines of a roof truss. Find the angle C and the length of the rafters a and b.

6. In Fig. 167 find the radii R_1 and R_2 for laying out the three centers A, B, and C so that they may be located as shown.

7. In Fig. 168 the diameter of a boiler is given as 76¾ in. and angle "A" is 150°. Find the length of water line "x."

8. It is desired to find the distance between two points A and B on opposite sides of a river. If the distance from B to a point C, Fig. 169, is found to be 128 ft. and the angles ABC and BCA are

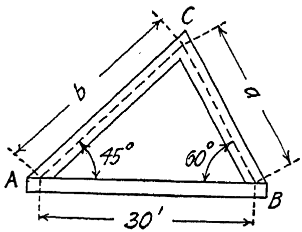


FIG. 166.

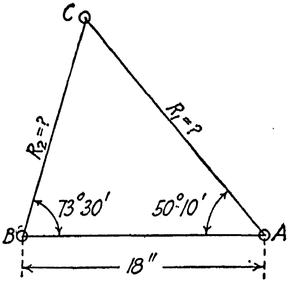


FIG. 167.

found to be 43° 40′ and 50° 20′ respectively, what is the distance AB?

9. It is desired to find the height of the stack shown in Fig. 170. The angle of elevation of a point in the horizontal plane

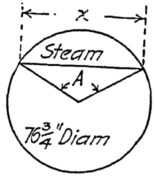


FIG. 168.

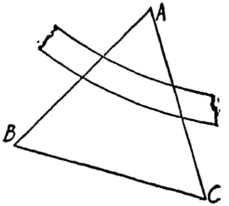


FIG. 169.

of the base of the stack is 54° 30′, and the angle of elevation at a point 50 ft. farther away is 44° 20′. Find the height of the stack.

10. The angle of elevation of the top of a stack at a point A, as shown in Fig. 171, is 69° 20′ and the angle of elevation at a point B, 50 ft. farther away, is 54° 50′. If the angle of elevation at

point A of the top of the building is 45° 20′, find height of stack and height of building.

11. Two trains start from the same station at the same time traveling on tracks which intersect at an angle of 64° 20′. If the

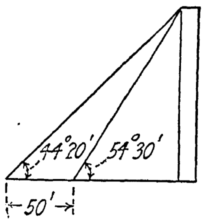


FIG. 170.

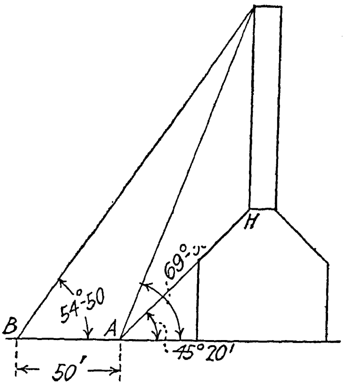


FIG. 171.

trains travel at the rate of 35 and 50 m.p.h., respectively, how far apart are they at the end of 45 min.?

12. A ¼-in. steel plate has the form of a parallelogram as shown in Fig. 172. If the sides are 64 in. and 40 in. in length and the

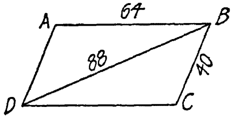


FIG. 172.

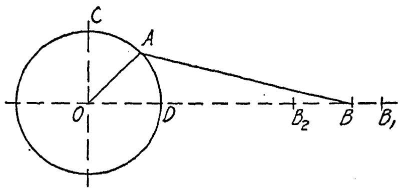


FIG. 173.

longer diagonal is 88 in. in length, find the angles A, B, C, and D, the area of the plate, its volume, and also its weight if 1 cu. in. of the metal weighs .283 lb.

13. In a steam engine the crank OA (Fig. 173) is 13 in. long, and

the connecting rod AB is 125 in. long. How far has the center B of the cross-head moved from its head end dead center position B_1 when the crank pin is midway between C and D?

14. Given a ½-in. steel plate with the shape as shown in Fig. 174. Find its area in sq. in. and its weight if steel weighs .28 lb. per cu. in.

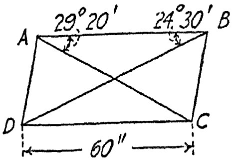


Fig. 174.

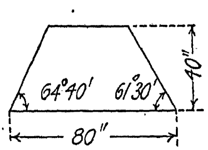


Fig. 175.

15. Given a ⅜-in. steel plate as shown in Fig. 175 with its longest side 80 in. and width 40 in. Find the non-parallel sides, its area in sq. in., and its weight.

CHAPTER XIX

ELECTRICITY

68. Definitions and Calculation of Resistance.—In making electrical calculations we need to understand the meaning of electrical "*resistance*" and how to calculate it. Every wire or electric circuit or portion of a circuit which conducts an electric current offers a "*resistance*" to the flow of the current through it. We may consider this *resistance* in a way like mechanical friction. Less water will flow in a pipe that is rough than in a smooth pipe because friction cuts down the flow. Less electric *current* will flow in a wire or "conductor," as it is called, having a high *resistance* than in a low-resistance conductor. The effect of the electrical resistance is to cause a part of the electrical energy to be changed into heat. Every electrical conductor is therefore heated by an electric current flowing through it. The less the resistance of a conductor the less the heat produced by a given current. Electrical resistance is measured in *ohms*.

The resistance of a conductor depends on its material, that is, whether copper, aluminum or iron, upon its length, area of cross section and its temperature. Copper is a better conductor of electricity than either aluminum or iron, that is, it offers less resistance to the flow of the electric current through it. If copper wire is heated its resistance increases. The resistance of a wire increases as its length is increased and decreases as its area of

cross section is increased. In electric wiring it is necessary to have wire large enough in cross section to carry the current required without overheating. If a wire too small is used there is danger from fire and insurance rules limit the size of wire for different kinds of work. In calculating the resistance of wires of circular cross section, their area is often found in *circular mils*, by squaring the diameter expressed in thousandths of an inch. Thus a wire $\frac{1}{4}$ in. or .25 in. in diam. has $(.25 \times 1000)^2$ or 62,500 circular mils in it. This is not the area as we determine the area of a circle, but is a convenient way of expressing relative areas of wire cables, etc. To find the resistance of copper wire at ordinary temperature we use the following rule:

$$\text{Resis. in ohms} = \frac{10.8 \times \text{length of wire in ft.}}{\text{area of cross section in cir. mils}}$$

Using this rule the resistance in ohms of 1 mile (5280 ft.) of copper wire .12 in. in diam. is equal to

$$\frac{10.8 \times 5280}{(.12 \times 1000)^2} \text{ or } \frac{10.8 \times 5280}{120 \times 120} = 3.96 \text{ ohms}$$

For another example, the resistance of 2000 ft. of copper wire .20 in. in diam. is

$$\frac{10.8 \times 2000}{(.20 \times 1000)^2} \text{ or } \frac{10.8 \times 2000}{40,000} \text{ or } .54 \text{ ohm.}$$

Ten miles of a 100,000 circular mil copper cable has a resistance of

$$\frac{10.8 \times 10 \times 5280}{100,000} \text{ or } 5.70 \text{ ohms}$$

69. Resistances in Series. If a number of electrical conductors are arranged one after another in a circuit as in Fig 176 so that the same current flows through all of

them, they are said to be "in series" and their total resistance is equal to the sum of their separate resistances:

Total resistance = resistance of A + resis. of B + resis. of C

FIG. 176.

70. Resistances in Parallel.—If the conductors are arranged side by side as in Fig. 177 so that the current divides in going through them and each conductor takes only a part of the total current, they are said to be "in parallel" and the *reciprocal* of the total resistance is equal to the sum of the reciprocals of the separate resistances:

$$\frac{1}{\text{Total resistance}} = \frac{1}{\text{resis. } A} + \frac{1}{\text{resis. } B} + \frac{1}{\text{resis. } C}$$

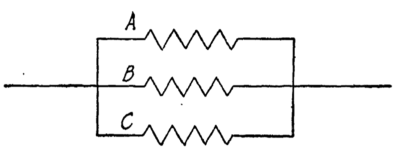


FIG. 177.

For example, if a copper cable of .10 ohm resistance is in parallel with an aluminum cable of .85 ohm resistance the combined resistance is found as follows:

$$\frac{1}{\text{total resistance}} = \frac{1}{.10} + \frac{1}{.85}$$

or

$$\frac{1}{\text{total resistance}} = 10 + 1.18 \text{ or } 11.18$$

$$\text{and total resistance} = \frac{1}{11.18} \text{ or } .0895 \text{ ohm.}$$

For transmitting power, copper, aluminum, steel, and phosphor bronze wire are used, copper and aluminum principally. For telephone and telegraph work steel and iron wires are used. For protective devices such as fuses and for coils of electrical measuring instruments, various alloys or mixtures of metals are used.

Copper is used to a large extent as an electrical conductor on account of its comparatively low resistance. Aluminum has an advantage over copper in its lighter weight for the same conducting power. This in transmission lines reduces the weight on insulators, poles, and cross-arms.

PROBLEMS ON RESISTANCE

1. Find the resistance of 5 miles of transmission line with conductors .187 in. in diam.

2. It is desired to extend the above line 2 miles with conductors .125 in. in diam. What is the resistance of the line added and of the total line?

3. Find the resistance of 12 miles of 150,000 circular mil copper cable.

4. An aluminum cable of 12 ohms resistance is put "in parallel" with the copper cable of Prob. 3. What is the resistance of the two in parallel?

5. A copper wire has a resistance of 20 ohms. Another wire of the same material is twice as long as the first and twice as large in cross section. How does its resistance compare with that of the first?

6. Give as many examples as you can of the use made of the heating effect of an electric current flowing through a resistance.

7. The resistance of a line wire is 4 ohms and a second wire of the same size is run along and "tied in" with the first, what is the resistance of the two taken together?

8. The resistance of a copper wire is 5 ohms. What is the resistance of another wire of the same material three times as long and of one-third the cross-sectional area of the first?

9. What metals are used as electrical conductors besides copper and aluminum? Explain in what kind of service they are used.

10. From the table on this page what number wire would you use for concealed work to carry a current of 22 amperes? What part of an inch is the diameter of this wire, and what does it weigh per mile if insulated?

71. Safe Carrying Capacity of Wires.—

Gauge No., B. & S.	Diameter, mils.	Ohms per 1000 ft.	No. amperes, open work	No. amperes, concealed work	Lb. per 1000 ft., bare	Lb. per 1000 ft., insulated
18	40	6.3880	5	3	4.92	18
17	45	5.0660	6	4	6.20	21
16	51	4.0176	8	6	7.82	25
15	57	3.1860	10	8	9.86	31
14	64	2.5266	16	12	12.44	38
13	72	2.0037	19	14	15.68	43
12	81	1.5890	23	17	19.77	48
11	91	1.2602	27	21	24.93	64
10	102	.99948	32	25	31.44	80
9	114	.79242	39	29	39.65	97
8	128	.62849	46	33	49.99	116
7	144	.49845	56	39	63.03	118
6	162	.39528	65	45	79.49	166
5	182	.31346	77	53	100.23	196
4	204	.24858	92	63	126.40	228
3	229	.19714	110	75	159.38	265
2	258	.15633	131	88	200.98	296
1	289	.12398	156	105	253.43	329
0	325	.09827	185	125	319.74	421
00	365	.07797	220	150	402.97	528
000	410	.06134	262	181	508.12	643
0000	460	.04904	312	218	640.73	815

72. Ohm's Law, Calculation of Current, Voltage and Resistance.—In order to force an electric current through

a wire or conductor in spite of its resistance it is necessary to have an electric pressure just as it is necessary to have water pressure or "head" to force water through a pipe in spite of friction.

Electric pressure is measured in *volts* and is indicated by an instrument called a *voltmeter*. Instead of electric pressure we sometimes speak of the "voltage" of a circuit or the "electromotive force" of the circuit. The three terms "electric pressure," "voltage," and "electromotive force" all mean the same pressure required to force an electric current through a circuit. The amount of current which will flow in a circuit of given resistance and under a given pressure is found from the following rule known as Ohm's Law (named from Dr. Ohm):

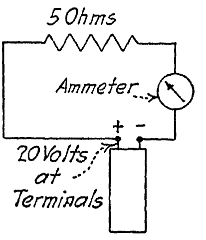


FIG. 178.

$$\text{Current} = \frac{\text{voltage}}{\text{resistance}}$$

If the voltage is in "volts" and the resistance in "ohms" the current is in "amperes." For example, if, as in Fig. 178, a 20-volt battery is connected to a wire of 5 ohms resistance a current of $\frac{20}{5}$ or 4 amperes will flow through the circuit, and may be read on the "current meter" or "ammeter."

For example, if an incandescent carbon filament lamp having a resistance of 220 ohms is put on a 110-volt circuit, the current which it will take equals $\frac{110}{220}$ or .5 ampere. From the rule that

(1) Current $= \dfrac{\text{voltage}}{\text{resistance}}$ it follows that

(2) Voltage = current $\times$ resistance, and also

(3) Resistance $= \dfrac{\text{voltage}}{\text{current}}$.

Rule (2) enables us to find the voltage across a known resistance through which a known current flows, and rule (3) enables us to find the resistance of a circuit with a known current flowing under a known pressure.

In Fig. 179, three lamps are arranged *"in series"* across 110-volt mains. Here the *current through each*

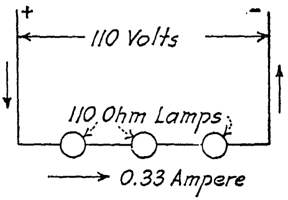


FIG. 179.

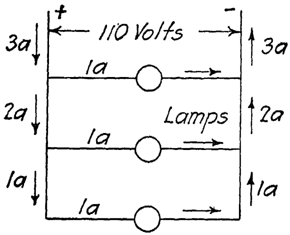


FIG. 180.

lamp is the same and if each lamp has a resistance of 110 ohms, the three "in series" have a resistance of 3 $\times$ 110 or 330 ohms. The current through the lamps therefore equals $\dfrac{\text{the voltage}}{\text{the resistance}}$, or $\dfrac{110}{330}$ or .33 ampere. From rule (2) the voltage across each lamp equals current $\times$ resistance, or .33 $\times$ 110 or 36.7 volts.

Fig. 180 shows three 110-ohm lamps arranged *"in parallel"* across 110-volt mains. The current through each lamp equals $\dfrac{\text{the voltage}}{\text{resistance of lamp}} = \dfrac{110}{110}$ or 1 ampere. The three lamps together therefore take three amperes.

The arrows indicate the direction of flow of the current and the figures, the number of amperes in the different parts of the circuit.

PROBLEMS

11. What is the resistance of a magnet winding through which 1.5 amperes of current flows under a pressure of 110 volts? Draw a figure showing the magnet in circuit?

12. If a 32-c.p. carbon lamp of 110 ohms resistance is connected across a 110-volt circuit, what current flows through it? How much current do ten of these lamps take when arranged "in parallel" on a 110-volt circuit?

13. How much current would each of ten 220-ohm lamps take if put "in series" in a 550-volt circuit?

14. A carbon filament lamp operating on 110 volts takes .95 ampere when first put in circuit and .93 ampere after burning for 10 min. What is the change in resistance due to heating the filament?

15. Three 110-ohm lamps are connected in series and these in series with a *parallel* group of five 220-ohm resistances. What is the resistance of the total number of lamps as arranged?

16. In Prob. 15 what is the voltage across each part of the circuit when 110 volts are impressed on the total circuit described?

17. Two thousand feet of No. 18 B. & S. gauge copper wire is to be wound on a magnet and operated on a 110-volt circuit. What resistance must be put "in series" with this winding in order that the current may be kept within safe limits? (See table, page 194.)

18. What is the weight of 10 miles of No. 00 B. & S. gauge trolley wire?

19. What is the total current taken by ten lamps in parallel if each has a resistance of 121 ohms and operates on a 110-volt circuit?

73. Calculation of Power and Energy in an Electric Circuit. The *power* consumed in a *direct current* circuit and the *power* taken by an incandescent lamp, whether on a *direct* or *alternating* current circuit equals the product

of the current flowing through the circuit and the voltage across the circuit, that is,

$$\text{power} = \text{current} \times \text{voltage}$$

When the current is in "amperes" and the voltage in "volts," the *power* is in *watts.*

For example, if an incandescent lamp, as in Fig. 181, across 110-volt mains and has 1 ampere flowing

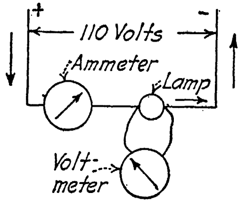


FIG. 181.

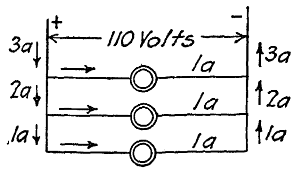


FIG. 182.

through it, the *power* in watts equals the voltage 110 multiplied by the current 1 or watts = 110 × 1 or 110.

In Fig. 182, three 110-ohm lamps are shown in *parallel* across 110-volt mains. By the rule for finding the current, we have the current taken by each lamp =

$$\frac{\text{the voltage across lamp}}{\text{resistance of the lamp}} \quad \text{or} \quad \frac{110}{110} = 1 \text{ ampere.}$$

The *power* taken by each lamp = current through lamp × voltage across lamp = 1 × 110 or 110 *watts.* The three lamps together therefore take 110 × 3 or 330 *watts.* The arrows and numbers in the figure show the direction of flow of the current and the number of amperes in each part of the circuit.

One thousand watts is called a *kilowatt.* In the

example above 330 watts or $\frac{330}{1000} = .330$ of a kw. is the *power* consumed constantly.

The *energy* consumed in an electric circuit equals the product of the *power* and the *time* during which the *power* is taken. If the *power* is in *watts* and the *time* in *hours*, their product gives the *energy* in *watt-hours*. If the *power* is in *kilowatts* and the *time* in *hours* their *product* gives the *energy* in *kilowatt-hours* which is the unit used by electric light and power companies. If lights which take .30 kw. are kept burning for 3 hr. the *energy* they consume in this time equals kilowatts $\times$ hours or $.30 \times 3$ or .90 of a kw.-hr.

Electrical energy is sold at prices ranging from a fraction of a cent to 18 or 20 cts. per kw.-hr. (kilowatt-hour). At 8 cts. per kw.-hr. the cost of the energy consumed by the lamps taking .90 kw.-hr. is $.90 \times 8$ or 7.2 cts.

PROBLEMS

20. What power is taken by a 110-volt carbon filament lamp with a current of .98 ampere?

21. A tungsten lamp takes .545 ampere at 110 volts. What is its wattage?

22. What is the difference in power taken by a group of five 110-volt carbon filament lamps each taking 1 ampere and a cluster of three 110-volt tungsten lamps each taking 60 watts?

23. Calculate the kilowatt-hours of energy consumed in 5 hr. by three 100-watt tungsten lamps.

24. How much energy is taken in 2 hr. by twelve 60-watt lamps?

25. What is the cost per month of 30 days of 2 hr. of lighting per day, for an entire building with energy of 8 cts. per kw.-hr., if there are twelve 100-watt lamps and the equivalent of twenty 40-watt lamps in the building?

26. Find the cost per month (30 days) for house lighting, with

4 carbon filament lamps each taking 1 ampere at 110 volts and burning for an average of 5 hr. per day. Take the cost of energy at 8 cts. per kw.-hr.

27. What kinds of incandescent lamps are in general use? State the advantages and disadvantages of each.

28. An office is first lighted with six 16-c.p. carbon filament incandescent lamps taking 3.1 watts per candle. The carbon lamps are later replaced by four 60-watt tungsten lamps. What is the saving per month of 30 days with the lamps burning 5 hr. per day if energy costs 8 cts. per kw.-hr.?

29. How does the first cost of carbon and tungsten lamps compare? Is there a saving in using tungsten rather than carbon lamps?

74. Electrical Horsepower.—*Horsepower* means the rate at which work is done. For example, 33,000 ft.-lb. of work done per min. or 550 ft.-lb. per sec. represents *one horsepower*. In electrical terms one horsepower (abbreviated h.p. or HP) equals 746 *watts*. To find the horsepower taken by a direct-current motor multiply the voltage on the motor by the current taken by the motor and divide the product by 746, that is

$$\text{horsepower of a motor} = \frac{\text{Volts on motor} \times \text{current intake}}{746}$$

Example.—Find the horsepower consumed by a 110-volt motor taking a total current of 10.3 amperes.

$$\text{Horsepower} = \frac{110 \times 10.3}{746} \text{ or } 1.52 \textit{ Ans.}$$

To find the horsepower delivered by an electric *generator*, multiply the voltage at the generator terminals by the *current output* and divide the product by 746, that is

$$\text{horsepower of a generator} = \frac{\text{Volts at gen. ter.} \times \text{curr. out.}}{746}$$

Example.—An electric generator with 110 volts at its terminals supplies 25 amperes to a lamp board. What horsepower is it giving out as useful power?

$$\text{Horsepower} = \frac{110 \times 25}{746} \text{ or } 3.69 \quad \textit{Ans.}$$

To find the *horsepower-hours*, multiply the load in horsepower by the time in hours during which the power is supplied.

Example.—A generator supplies 13 amperes at 120 volts for 5 hr. what is its horsepower hour output?

$$\text{Horsepower hours} = \frac{120 \times 13}{746} \times 5 = 10.45 \quad \textit{Ans.}$$

PROBLEMS

30. What horsepower generator is necessary to supply 30 amperes at a pressure of 110 volts?

31. A motor driving a lathe takes 6.4 amperes at 220 volts, what is the horsepower supplied to the motor?

32. Three direct-current and voltage ratings are as follows: 10 amperes, 110 volts; 15 amperes, 220 volts; 30.5 amperes at 110 volts. Find the corresponding horsepower rating of each of these motors when working at full load.

33. Five tungsten lamps, each take .95 ampere at 110 volts, what is the number of watts and horsepower taken by the lamps? If the lamps burn for 5 hr. what energy do they take in watt-hours, kilowatt-hours, and in horsepower-hours?

34. What kind of instruments are used for measuring an electric current? Find out over what range these instruments are made to read.

35. What kind of instruments are used for measuring electric pressure or voltage? Find out what range these instruments have.

36. How are electrical "power" and "energy" measured?

37. Find out what different kinds of electric lamps are in general use, with the advantages and disadvantages of each.

38. What rules do you know in regard to personal safety and protection of machines when working about electrical circuits?

75. Methods of Charging for Electrical Energy.—(*a*) FLAT RATE SYSTEM.—In the flat rate system the customer is charged a certain amount per month for each lamp installed. No meters are used to determine the amount of energy consumed. With this system a wasteful consumer pays no more for the same number of lights installed than an economical user. This system saves the cost of meters, their repair, and simplifies the company's bookkeeping. A scale of discounts is usually arranged increasing with the number of lamps installed.

(*b*) METER SYSTEM.—In this system the consumer is charged for the energy which he uses, that is, for the number of kilowatt-hours which he consumes in a certain time. This method would seem to be a just way of charging for energy, but the time of day that power is taken is of as much importance as the amount taken. Power taken at a time when the station load is heavy is more expensive for the company than power taken at a time of light load. The cost of power depends on the size of the power station, the amount of power taken, and the time at which it is taken. In view of these facts a "two rate" meter is sometimes used with two sets of dials, one reading at times of light station load and the other at times of heavy station load. The consumer is then charged one rate for the power taken when the station is working light and a higher rate for power taken at times of heavy load on the station.

A "maximum rate" meter is also used in some cases. This meter gives the greatest load or power which the consumer has taken during a certain period. The charge is then made on a basis of the greatest amount of power taken at any one time as well as on the total number of kilowatt-hours consumed.

A certain charge is usually made whether the customer uses any power or not. This is because the customer has power at hand any time he wishes to use it, and a charge is made for this service.

PROBLEMS

39. Find the total resistance of the circuit shown in Fig. 183.

40. If the circuit of Prob. 39 is connected across 110-volt mains, what is the current through the circuit and the voltage across each portion of the circuit?

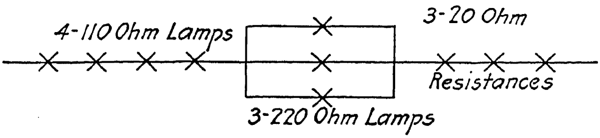


FIG. 183.

41. Find the difference in power taken by eight 220 ohm lamps operating on a 220-volt circuit and sixteen 100-watt lamps.

42. If a current of 8.2 amperes flows through a resistance on 110 volts, how much more resistance must be put in circuit to reduce the current to 4.5 amperes?

43. How many horsepower-hours are equivalent to 2000 kw.-hr.? How many kilowatt-hours are consumed by a lamp load of 125 amperes at 220 volts in circuit for 4 hr.?

44. Calculate the power in watts taken by thirty 110-volt carbon lamps connected in parallel if each lamp takes 1.5 amperes.

45. Calculate the power taken by 20 lamps in series on a 1100-volt circuit if 15 amperes flow through the circuit.

46. Calculate the power in kilowatts taken by ten 110-volt lamps in parallel across a 110-volt circuit if each takes .9 ampere.

47. Ten lamps taking 1 ampere each are connected in parallel across 110-volt mains. What is the resistance of each lamp and the power taken by the circuit?

48. Four 110-volt carbon lamps taking 1 ampere each were replaced by four 40-watt tungsten lamps. If the cost of energy was 8 cts. per kw.-hr., how much did they gain or lose per hour by the change?

49. Find the difference in kilowatt-hours taken in 5 hr. by ten 220-ohm lamps on 110-volts and eight 60-watt tungstens.

50. Find cost of operating the circuit shown in Fig. 184 per month of 30 days and 4 hr. per day, energy 10 cts. per kw.-hr.

51. What system of charging for electricity do you consider the best, and why?

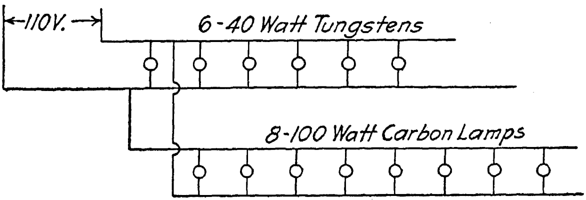


FIG. 184.

76. Comparison of Water and Electric Systems.—A very close and interesting comparison can be made between a water system with its piping, gauges, pumps, motors and so forth and an electric system with its wiring, meters, generators, motors and so forth. In Figs. 186 and 187 diagrams are shown of typical water and electric systems, with the corresponding parts in the two systems marked and explained as follows:

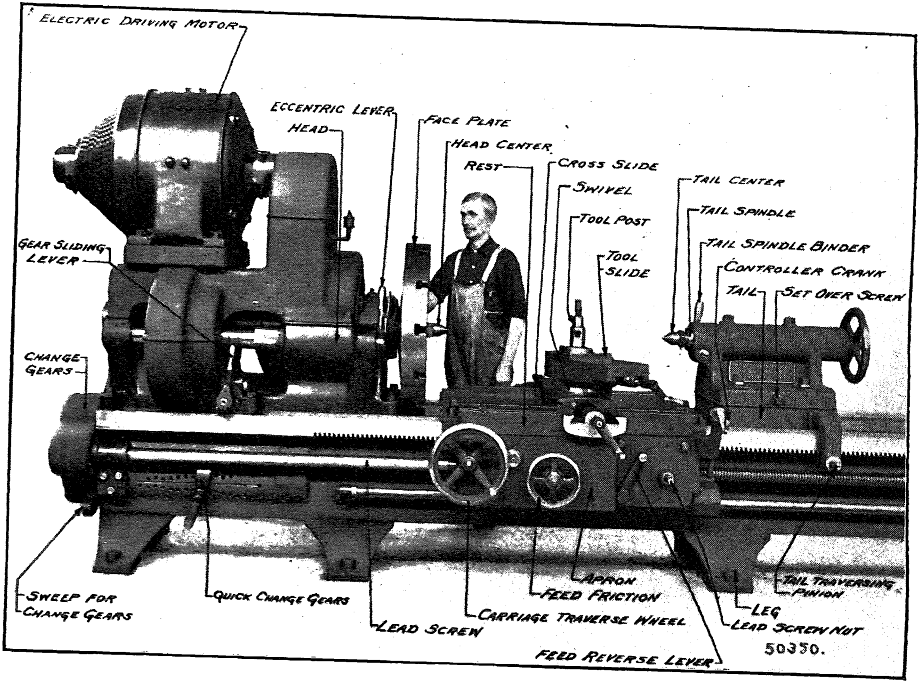


FIG. 185.—Cut showing a lathe with an electric motor drive.

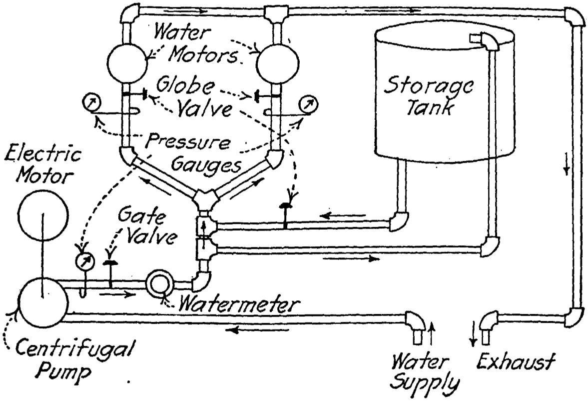


FIG. 186.

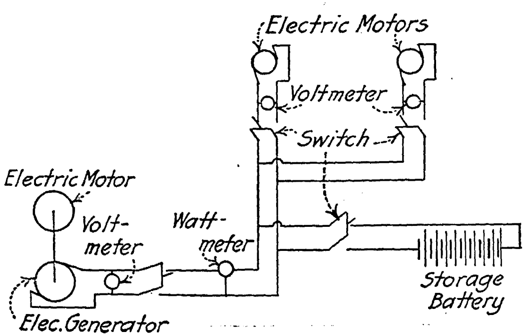


FIG. 187.

Water system	Corresponding part in electric system
Electric motor for driving centrifugal pump.	Electric motor for driving electric generator.
Centrifugal pump.	Electric generator.
Pressure gauge, giving water pressure.	Voltmeter giving electric pressure.
Gate valve for shutting off water.	Switch for opening electric circuit.
Water meter for measuring quantity of flow of water.	Wattmeter for measuring the power taken by the circuit.
Globe valve for shutting off water supply to motor.	Switch for cutting off electric current to electric motor.
Water motor for doing mechanical work.	Electric motor for doing mechanical work.
Storage tank for water pumped by centrifugal pump.	Storage battery for electricity supplied by generator.

CHAPTER XX

STRENGTH OF MATERIALS

77. Definition.—All materials have a property or characteristic which we call "strength," which is quite different for different materials such for example as wood and steel. In order to know what kind of material to use for a particular piece of work and how much of it to use, that is how large a piece is required in order that the completed work or job shall be sufficiently strong to serve its purpose, it is necessary to know what the strength of the various materials is and how this strength differs for the different methods in which pressure may be brought on the material.

The average person is likely to speak only relatively regarding the strength of materials, namely that one material is so many times stronger than another. It is necessary, however, in working with machines and in general construction work to have more definite knowledge of the strength of materials. This has led to the construction of *testing machines*, capable of measuring directly in pounds the strength of the materials under various conditions. These machines are described later on in this chapter.

The object of this chapter is to convey some idea of the strength of materials, from results secured by actual tests and how these results are used in calculating the size and amount of material required.

This chapter also brings out some of the effects of forces applied to materials in changing their size and shape, and also in merely producing pressure in them without changing their form.

78. Stress and Strain.—Whenever a *pressure* or a *load* is put upon any piece of material it tends to change the shape of the piece. The material itself resists this change of shape and in so doing exerts a force *opposite* to the load or pressure applied to the material. For example, if we take a piece of string and, holding it in our hands, pull on it with an increasing force the string will finally break, but not until it has straightened out and become taut. After this there is a gradual stretch until the fibers pull apart. Our effort or pull on the string has been opposed by a resistance in the fibers of the string. This resistance is called *stress*. *Stress* may therefore be defined as the resistance which a material offers to the action of an external force, pressure or load which tends to change the shape of the material.

If a 2-ton weight is being lifted by a rope the *stress* produced in the rope is 4000 lb.

The stretching of the string lengthened or *elongated* it and produced a change of its form. This change of form in the fibers of a material is called *strain*.

In construction work the materials used must be of sufficient size and strength so that the loads applied are not sufficient to change their shape, otherwise the parts are likely to break or "rupture" and disastrous results follow.

79. Kinds of Stresses.—There are three general kinds of stresses, namely (1) Tensile stress (or pulling stress), (2) Compressive stress (pushing or crushing stress), (3) Shearing stress (cutting stress).

There are two other kinds of stresses which in reality are combinations of those given above. These are (*a*) *Torsion* or *twisting* which is a kind of shearing stress

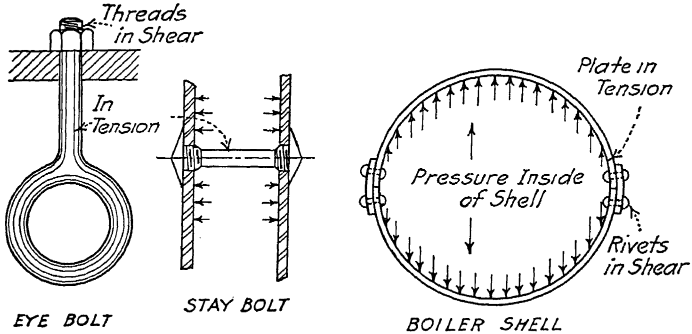


Fig. 188.

and (*b*) *Bending* or *flexure* which is a combination of tension and compression with or without shearing. The examples (Figs. 188, 189 and 190) show the various stresses of materials in service.

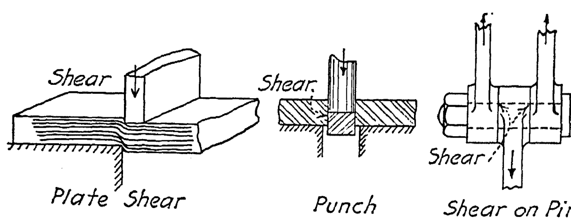


Fig. 189.

Since the tensile stress set up in a material is due to a pull, the *tensile strength* of the material is a measure of the resistance offered by its fibers to being pulled apart.

Since the compressive stress is a stress set up in a material due to a push, thrust, or crushing force, the *compressive strength* is a measure of the resistance offered by the fibers of the material to being crushed.

A *shearing stress* is one set up in a material due to a force acting at right angles to the material and the shearing strength is a measure of the resistance of the fibers of the material to being cut or sheared off.

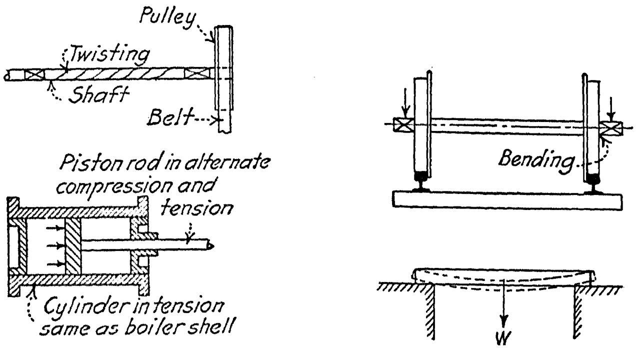


FIG. 190.

80. Ultimate Strength.—If samples of different materials are loaded until they break or rupture we can find out how much each material will stand. The *ultimate strength* of a material is the number of pounds per square inch of cross section of the material at which it breaks. This value varies between wide limits for different materials and often varies considerably for the same material.

The ultimate strength of a given material is different for the different stresses of tension, compression and shear.

81. Testing Machines.—For testing materials to find their ultimate strengths, testing machines as shown in the following figures are used.

Fig 191 shows a machine used for *tension* and *com-*

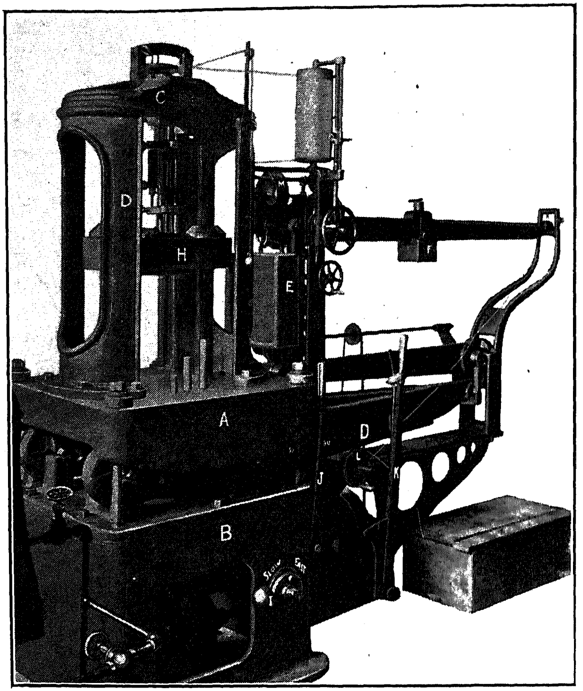


FIG. 191.

pression tests. The figure shows a piece being tested in tension.

The test specimen is placed in the machine as shown between heads (H) and (C) and is held firmly in place by means of wedge grips. Head (C) is fixed to the body

of the machine (A) by frame (D). The bed (A) rests on a set of levers. Through the head (H), two long screws are threaded which are geared to bevel gears shown inside of the case (B). A belt connected to a pulley which is fastened on a shaft to which the gears are fastened, drives the machine, which causes the screws to revolve slowly. This action moves (H) down causing a pull on the test piece and hence on the head (C), the pull being transmitted to (A).

(A) resting on lever (D) forces (D) down and by the system of levers, causes the lever on which (F) rests to move up. The system of levers is kept balanced by

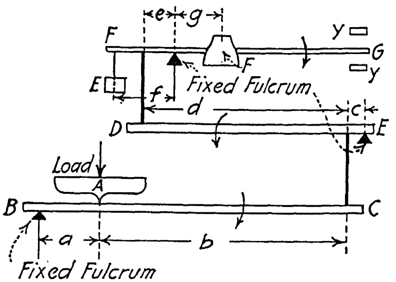


Fig. 192.

moving the weight (F) out along the graduated bar. (E) and (M) are weights used for balancing and for adjustment. All readings of the load applied and also the breaking load, are registered by means of (F), the lever on which it moves being marked off in pounds so as to read directly the pull exerted on the test piece.

When a piece is to be tested in compression, it is placed between the head (H) and (A) and it can be seen that as the head (H) moves down, the piece is compressed and the load registered as before.

The system of levers is shown in Fig. 192 giving an outline of their arrangement.

As the load is applied to the test specimen, it pushes down on (*A*) and tends to move lever (*BC*) around the fixed point at (*B*). The force at (*C*) then pulls down on the lever (*DE*) which is pivoted at (*E*). The lever (*FG*) is connected to the lever (*DE*) by the link (*FD*),

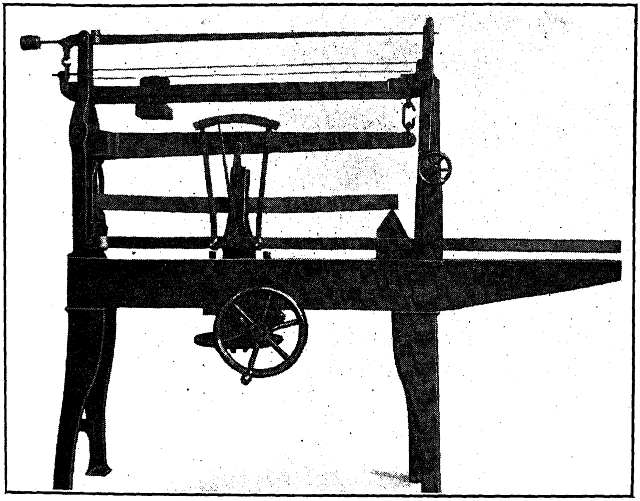


FIG. 193.

and as the force at (*D*) pulls down on the end (*F*) of lever (*FG*), the end (*G*) tends to move up. The end (*G*) is kept balanced between (*Y*) and (*Y*) by moving the weight (*F*) out along the lever. The object of the system of levers is merely to reduce the large load acting at (*A*) to a small load acting at (*F*). Nearly all testing machines are built on the principle of a system

of levers and the methods are worked out in a manner practically similar to that just described.

For bending tests of wood specimens a machine similar to that shown in Fig. 193 is used. The lever arrangement is similar to that of the tension machine. The method of supporting the specimen and recording the load on the beam and the deflection of the beam can be readily seen. This machine is operated by hand.

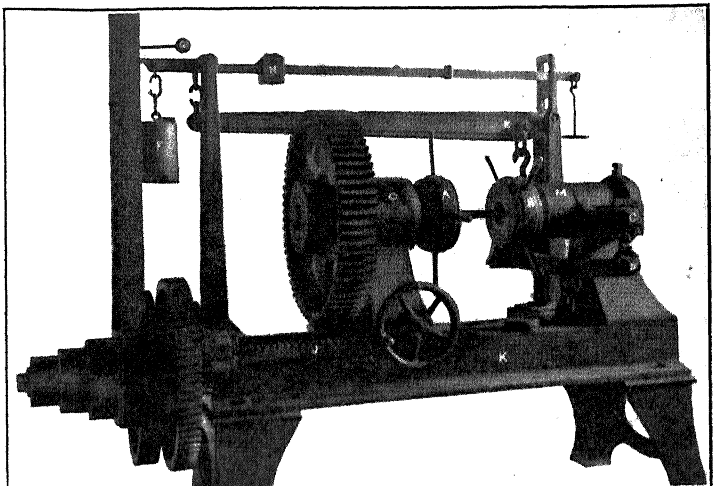


FIG. 194.

Pieces to be tested in torsion are placed in a machine similar to that shown in Fig. 194. The specimen is fastened in grips (*A*) and (*B*). (*B*) is fastened to (*M*) which has an arm (*C*) attached to it. By means of the belt shown at the left and the cone pulley and system of gears, (*A*) can be rotated. As it moves it tends to turn (*B*) and (*M*) and (*C*) tends to rise. Arm (*C*) then

pulls up on (*D*) which is a lever pivoted to the frame and so that the force is carried to the lever on which (*H*) rides. The lever is kept balanced by moving weight (*H*). The weight (*F*) serves as a balance. The system of levers here shown is practically the same as for the machine shown in Fig. 193 and the method of recording the load is similar.

82. Standard Test Pieces.—Figs. 195, 196 and 197 show a few of the standard test pieces or specimens used in finding the strength of materials.

The specimen used should in any case be as near an average specimen of the material as possible. If a

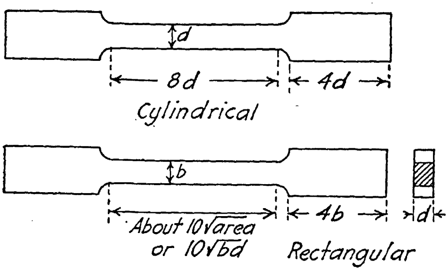


FIG. 195.

shipment of material is to be purchased, a few samples should be taken from different parts of the supply.

Fig. 195 shows a cylindrical and rectangular section of iron or steel as used in a *tension* test. The pieces are machined all over and filed along the middle part, for the body of the material should be as free from tool marks as possible and of a uniform size. The proportion of the parts is given in terms of the cross section of the material.

The blocks shown in Fig. 196 represent about the

limit of the size of pieces used in compression tests. A piece should not be longer than 6 times its diameter, else a bending may occur along with the compression.

The forms used for bending and shearing tests are like those shown in Fig. 197, the form in each case depending on the kind of material and the result to be determined. For torsion tests, the specimens are like those used for tension tests.

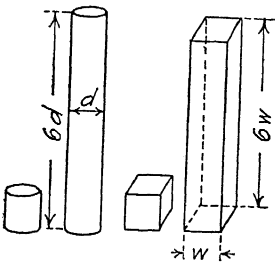


FIG. 196.

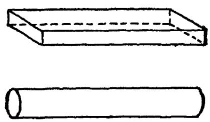


FIG. 197.

83. Values of Ultimate Strengths.—From the results of tests with these machines the following table shows the average values of ultimate strengths of the most used materials and for the kind of stress indicated.

ULTIMATE STRENGTH IN POUNDS PER SQUARE INCH

Material	Tension	Compression	Shear
Cast iron........	20,000	90,000	20,000
Wrought iron....	55,000	50,000	40,000
Average steel....	60,000	70,000	50,000
Soft copper......	25,000	40,000	20,000
Hard copper.....	40,000	55,000	40,000
Wood..........	10,000	8,000	3,000 (across grain)

The following results represent a fair average

Material	Tension	Compressor	Shear
Granite..........	....	15,000	1,500
Limestone.......	....	12,000	1,400
Concrete........	....	2,000	1,000
Brick...........	....	5,000	1,000

It would be of course unsafe to use in actual service material under such conditions that it would be subjected to a stress near the ultimate stress of the material, since the piece might break at any moment. A piece of material put in actual service is therefore designed for a stress per square inch, which is much less than the ultimate stress. This stress is called the *working stress* or the *safe working stress.*

84. Factor of Safety.—The ultimate strength divided by the working stress is called the *Factor of Safety.* For example if a piece of steel breaks at 80,000 lb. per sq. in. and we use a factor of safety of 8, the *working stress* is only $\frac{1}{8}$ of 80,000 lb. or 10,000 lb. per sq. in., and the design is worked out for the latter value. The factor of safety of a material varies according to the material used and the manner of loading. A much larger factor of safety is allowed for bridge members on which there is a varying load than for a building for example where the load is practically constant.

85. The Elastic Limit.—In designing structures such values of working stress are always taken that the material will not be stretched beyond its "*elastic limit,*" that is will not be worked at a unit stress beyond which the material will not return to its original shape when the stress is removed. The *Elastic Limit* which should always be greater than the working stress may also be defined as that unit stress beyond which the deforma-

tions of the material increase in a faster ratio than the applied loads, and beyond which the material will acquire a permanent set on the removal of the load, and not return to its original shape.

86. Values of Safe Working Stresses.—Following are safe working stresses in pounds per square inch for the most commonly used materials. The safe working load for tension, compression and shear are abbreviated respectively as follows S_t, S_c, S_s.

SAFE WORKING STRESSES—POUNDS PER SQUARE INCH

Material	Tension S_t	Compression S_c	Shear S_s
Cast iron........	4,000	12,000	3,000
Wrought iron....	10,000	12,000	10,000
Steel castings....	12,000	16,000	12,000
Timber.........	800	800	600 (with grain)

From the above we have the rule, the safe load "W" which can be carried for the several kinds of stresses is as follows:

For tension W = area in sq. in. $\times S_t$.
For compression W = area in sq. in. $\times S_c$.
For shear W = area in sq. in. $\times S_s$.

Or in general for all stresses

$$W = \text{area} \times S$$

from which $$\text{area} = \frac{W}{S}$$

These mean that the total safe load equals the stress per square inch times the area of cross section of the piece in square inches. And also the area required by

a piece equals the total load divided by the safe load per square inch.

From the calculated area, the length of a side of the cross section, or the diameter in case of a circle, and so forth may be found.

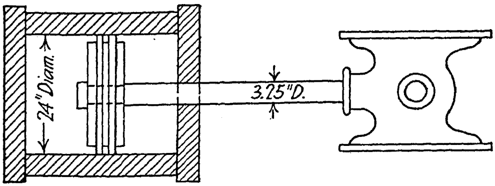


FIG. 198.

87. Strength of Rods.—When steam is applied to the back side of the piston shown in Fig. 198, the pressure puts the rod in tension. If the pressure is 180 lb. per sq. in. the total pressure on the piston is 180 times the net area of the piston, that is, subtracting the area of cross section of the rod. The area on which the pressure acts is shown cross sectioned in Fig. 199 and equals

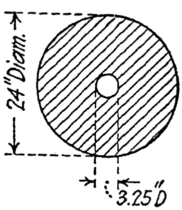


FIG. 199.

$$(24)^2 \times .785 - (3.25)^2 \times .785 \text{ or}$$
$$444.1 \text{ sq. in.}$$

The total pull on the rod therefore equals

$$444.1 \times 180 \text{ or } 79{,}900 \text{ lb.}$$

that is, the applied load is 79,900 lb. and the total resisting stress is therefore 79,900 lb. The stress per sq. in. on the rod equals

$$\frac{\text{total load}}{\text{area of rod}} \text{ or } \frac{79{,}900}{8.30}$$

which equals 9630 lb., and this is well within the safe limit for steel. The unit compressive stress is also within the safe limit for steel. The compressive stress occurs when steam is admitted to the head end of the cylinder.

Example.—Find the size of a circular steel rod from which a weight of 4 tons is to be suspended.

Solution.—Taking 14,000 lb. per sq. in. as the safe working load in tension, we have

$$\text{Area} = \frac{W}{S} \text{ or } \frac{4 \times 2000}{14{,}000} \text{ or } .572 \text{ sq. in.}$$

The diameter of the rod then equals

$$\sqrt{\frac{\text{area}}{.785}} \text{ or } \sqrt{\frac{.572}{.785}} \text{ or } .853 \text{ in.}$$

In figuring out threaded bolts the required area must be taken as that at the *root* of the threads, since a piece is no stronger than its weakest part. If the bolt has a sufficient strength for the area at the root of its thread, or in other words for its *smallest* area, the body of the bolt will have a surplus of strength on account of its greater area of cross section.

88. Strength of Ropes, Chains and Cables.—The ultimate strength of hemp rope is about 6000 lb. per sq. in. and for Manila rope about 3000 lb. per sq. in.

A safe working stress for hemp rope is 1400 lb. per sq. in. based on the *Nominal Area* of the rope, that is, assuming the area of cross section of the rope a solid circle. For Manila rope half the above value may be used.

The safe working load for chain links is given as 9000 lb. per sq. in. For iron cables the ultimate strength is about 40,000 lb. per sq. in. and for steel cables 80,000

lb. per sq. in. A factor of safety of 10 is often used for ropes and cables in finding the safe working stresses. This makes the safe value of working stress for iron cable 4000 lb. per sq. in. and for steel cable 8000 lb. per sq. in.

89. The Strength of Columns.—When a bar or rod subjected to compression has a length greater than 10 times its diameter it is called a *column*, and it must be worked out by difficult formulas which take account of its length, size and form of cross section. Columns may fail by buckling as well as being crushed, and most satisfactory results in some cases are obtained by testing the actual columns in a suitable testing machine. From the results of such tests the most reliable information on the strength of the columns is obtained.

PROBLEMS

1. It took 100,000 lb. pull in a testing machine to break a steel rod 1½ in. in diam. What was its ultimate tensional strength in lb. per sq. in.?

2. If a cast-iron bar 1½ in. by 2 in. in cross section breaks under a tensile load of 70,000 lb., what load in lb. per sq. in. would break a 2½-in. diam. cast-iron rod of equal unit strength?

3. A round cast-iron bar is to be subjected to a tension of 34,000 lb. If it is designed so that the unit stress is 2500 lb. per sq. in., find the diameter of the rod.

4. A wooden block 1⅝ in. square was crushed in a testing machine under a load of 21,640 lb. What was its ultimate compressive strength in lb. per sq. in.?

5. It required 87,630 lb. to shear off in a testing machine a 1¼-in. diam. steel rod. What was its ultimate shearing strength in lb. per sq. in.?

6. If the ultimate compressive stress of wrought iron is 55,000 lb. per sq. in. and it is subjected to a working stress of 11,500 lb. per sq. in., what is its factor of safety?

7. A ⅞-in. diam. wrought-iron tie rod on a bridge is subjected to a working stress of 9000 lb. per sq. in. What load is it sup-

porting? If the ultimate stress is 50,000 lb. per sq. in., what factor of safety is used in its design?

8. Find the compression in lb. per sq. in. for a 3-in. diam. piston rod carrying an 18-in. diam. piston with a steam pressure of 180 lb. per sq. in.

9. Find the tensile stress in lb. per sq. in. for a 3⅛-in. diam. piston rod with steam at 175 lb. per sq. in. acting on the back of a 20-in. diam. piston attached to the rod.

(NOTE.—Subtract the cross-sectional area of the piston rod from the piston area to get the *net* area on which the steam acts.)

10. What is the stress per sq. in. in the studs holding on a cylinder head for a 24-in. diam. cylinder if there are twenty-six

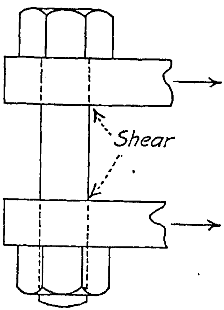


FIG. 200.

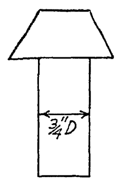


FIG. 201.

⅞-in. diam. studs and the greatest steam pressure is 200 lb. per sq. in?

(NOTE.—The area at root of thread of each ⅞-in. stud is .419 sq. in.)

11. A solid cast-iron cylinder 2½ in. in diam. is under a compression of 40,000 lb. Find its factor of safety.

12. A brick 2 in. × 4 in. × 8 in. weighs about 4½ lb. Find the height of a pile of bricks so that the compressive stress per sq. in. on the lowest brick will be ⅛ of the ultimate strength.

13. A wrought-iron bolt as shown in Fig. 200 is to be subjected to shear in two places. If the load is 3000 lb. and the factor of safety 5, find the diameter of the bolt.

14. What size of hemp rope is required to lift a casting weighing $2\frac{1}{2}$ tons?

15. What is the total safe load which a steel cable 2 in. in diam. can stand?

16. For an ultimate tensile strength of 50,000 lb. per sq. in. find the strength of the $\frac{3}{4}$-in. diam. cone rivet shown in Fig. 201.

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